Risk-minimising investment strategies — Embedding portfolio optimisation into a dynamic insurance framework

Received (in revised form): 15th January, 2011

Ursula Theiler
is president of the company Risk Training, a training and consultancy company in financial risk management. Formerly she gained several years of practical experience in both private and public German banks. She has spoken at various conferences and published diverse articles on topics in financial risk and integrated bank management. She holds a doctorate degree from the University of Munich (LMU), Germany and received different academic awards. She also contributes to national and international research projects.

Risk Training, Leite 37, D-14532 Kleinmachnow, Germany
Tel: +49 (0)170 3113101; E-mail: theiler@risk-training.org

Abstract While Markowitz's framework of portfolio optimisation aims to eliminate diversifiable risk, it does not consider protection against the undiversifiable, systematic risk of market downturns. This paper investigates a concept of risk-minimising investment strategies, which embeds revolving portfolio optimisations into a framework of dynamic portfolio insurance and thus links the two approaches of minimising the diversifiable and controlling the undiversifiable risk. Investment strategies are generated which minimise the diversifiable risk by revolving optimisations based on newer downside risk measures, value-at-risk and conditional value-at-risk. Portfolio insurance is applied on the investment strategies, in order to control the systematic risk over time. The concept is illustrated by a simplified application example based on historical data, which comprise the most recent period of the financial crisis. The example demonstrates that the performance of investment strategies is stabilised by the application of portfolio optimisation. The choice of risk measure turns out to be of minor relevance. In the period of market decline the impacts of portfolio insurance overlay the impacts of portfolio optimisation. The observations of the case study give reason to suggest that in times of financial instabilities portfolios primarily should be protected against downside risk by adequate portfolio insurance.

Keywords: *conditional value-at-risk, constant proportion portfolio insurance, dynamic asset allocation, portfolio insurance, time invariant portfolio protection, value-at-risk*

INTRODUCTION

The recent financial crisis hit many investors by surprise. Impacts of dramatic market downturns had been unexpected, when nearly all financial markets crashed in unison in a downward direction.

Extreme losses occurred owing to the lack of sufficient portfolio protection. The crisis also revealed that the application of modern portfolio optimisation by itself does not protect portfolios from suffering high losses, when systematic risks are occurring.

According to financial theory, risk can be decomposed into systematic, 'undiversifiable' risk on the one side and the issuer-specific, 'diversifiable' risk on the other side. While Markowitz's framework of portfolio optimisation aims to eliminate the specific risk, the model does not suggest any protection against the systematic risk of adverse market movements. On the other hand, concepts of portfolio insurance, also referred to under the notion of dynamic asset allocation, focus on portfolio protection against adverse market movements, comprising strategies that systematically adjust a portfolio's asset allocation, triggered either by changes inside the portfolio or in overall market conditions. A variety of strategies has been developed in this context in the last decades. Nevertheless, the link of those two streams of research, modern portfolio optimisation on the one hand and portfolio insurance on the other, has not been studied in detail so far.

This paper investigates a concept of risk-minimising investment strategies, which embeds revolving one-period portfolio optimisations into a framework of dynamic portfolio insurance and thus links the two approaches of controlling diversifiable and undiversifiable risk. Investment strategies are generated which minimise the diversifiable risk. Portfolio insurance is applied on the investment strategies, in order to control the systematic risk over time. The impacts of risk minimisation and portfolio insurance are analysed in a case study based on historical data, comprising the most recent period of the financial crisis. The case study aims to illustrate the suggested methodology. Impacts of minimising the diversifiable risk and of controlling the systematic risk are examined in a simplified portfolio setting.

In recent years the financial industry has extensively used quantile based downside risk measures. The risk measures value-at-risk (VaR) and conditional value-at-risk (CVaR) have become popular in finance. An intense discussion on the choice between VaR and CVaR has evolved in academic literature, concluding with mixed results. The question of the choice of risk measures is taken up in the applications presented in the case study. Investment strategies are generated based on risk minimisations in terms of the risk measures VaR and CVaR.

The paper is organised as follows. After the introduction in the second section the approach of risk-minimising investment strategies is described and a survey is given on the underlying conceptual framework. In the third section the case study is presented which consists of two steps. In the first step investment strategies are generated based on periodical VaR and CVaR minimisations. In the second step portfolio insurance is applied to the investment strategies derived from the first step and the performance of the different investment strategies is analysed. The final section of the paper concludes with both a summary and the prospect of further issues of research.

APPROACH OF RISK-MINIMISING INVESTMENT STRATEGIES

Survey

By definition, an investment strategy specifies a set of rules, behaviours or procedures, designed to guide an investor's selection of an investment portfolio.[1] Usually the strategy is designed around the investor's risk-return trade-off profile. In a dynamic context, portfolio management is a sequential process in which an investor revises his allocation periodically with respect to changing endogenous conditions and to the ongoing evolution on the financial markets. An investment strategy can therefore be represented as a series of asset allocations over time.

The paper suggests a concept of risk-minimising investment strategies, which embeds one-period portfolio optimisation into a framework of dynamic portfolio insurance and thus links the approaches of minimising the diversifiable and controlling the systematic risk. In order to minimise the diversifiable risk, at every rebalancing point of time a modified Markowitz optimisation problem is solved. By iteration over the entire investment period, a series of investment strategies is generated, which periodically minimise the specific risk. In order to additionally control the systematic risk, portfolio insurance is applied on the investment strategy over the investment period. Portfolio insurance, also referred to under the notion of dynamic asset allocation, comprises strategies that systematically adjust a portfolio's asset allocation, triggered either by changes inside the portfolio or in overall market conditions in order to maintain a specified floor level, the minimum target value.

The suggested concept of risk-minimising investment strategies is based on the following steps:

(1) At every rebalancing point of time a special rebalancing criterion is applied and a mean-risk portfolio optimisation is conducted in order to minimise the *diversifiable* risk.
(2) Over the investment period additional portfolio insurance is applied in order to limit the *systematic* market risk and maintain a specified floor level.

Figure 1 illustrates the author's basic idea of generating risk-minimising investment strategies.

In the following a survey is given on existing literature linked to the suggested approach. The underlying concepts of

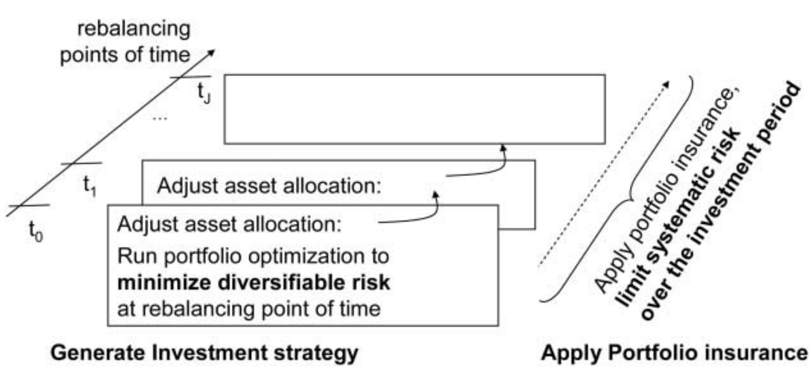

Figure 1: Concept of risk-minimising investment strategies

asset allocation go back to Markowitz's fundamental approach on portfolio selection.[2] The strengths and limitations of the one-period mean variance model since then have been widely discussed in the financial literature. Various extensions of the Markowitz model have been suggested to overcome difficulties of practical implementation, referring eg to a generalisation of the underlying assumptions or extensions to multi-period models.[3]

The traditional Markowitz model is based on the assumption that the asset returns follow a normal distribution. Yet frequently it turns out that this assumption does not hold in reality. A wide stream in the quantitative finance literature has discussed incorporating fat tails and skewness of the loss distributions into the traditional mean-risk allocation approach. Downside risk measures have been suggested in this context, thus generalising the underlying assumptions on the asset return distributions and the applied risk measure. In the recent past, the focus moved to newer quantile-based risk measures in this discussion.[4,5] The approach presented in this paper follows these ideas and the case study applies the risk measures value-at-risk (VaR) and conditional value-at-risk (CVaR) in a generalised mean-risk optimisation framework.

Another wide field of literature exists on the extension of the one-period Markowitz model into a multi-period setting. Pioneering work was suggested by Tobin using an application of discrete-time multi-period models. General frameworks for finding optimal decisions over the planning horizon evolved from this.[6] Stochastic optimal control theory was first suggested by Merton (1969) and taken on in a variety of studies.[7] The capital growth theory mainly goes back to Hakansson.[8] Diverse stochastic programming approaches were developed in more recent approaches.[9,10] Emmer *et al.* achieve optimal portfolio diversification in a time continuous setting, minimising the diversifiable risk in a long-term view.[11]

Nevertheless, multi-period optimisation problems often are difficult to implement owing to the underlying modelling assumptions or computational complexity. In this paper the aspect of a multi-period setting rather is considered by revolving one-period portfolio optimisations and embedding the series of asset allocations into a dynamic framework of portfolio insurance over time. In this way the diversifiable risk is minimised by revolving one-period optimisations and the systematic risk of adverse market movements is controlled over the investment period.

Dynamic asset allocation and trading strategies have been a matter of research for the last decades. Much attention was put on analysing the performance of trading strategies under different market conditions, without reaching a common consensus on the effectiveness of different portfolio insurance concepts. Studies have been conducted to optimise trading parameters and minimise transaction costs.[12] Different rebalancing criteria have been suggested for the revolving adjustment of asset allocations.[13] Nevertheless there has been little attention given to a comparison of using different rebalancing criteria. The application of portfolio optimisation and the use of new risk measures have been addressed recently in this context.[14] Yet they have not been closely examined from this view, which is the focus of this paper.

Definition of an investment strategy

In the following a formal description of an investment strategy is derived. Let the investment universe consist of n assets and let I represent the set of all asset weights $\mathbf{x} = (x_{1,} \ldots, x_n)'$, which add up to the normalised budget of 100 per cent:

$$I = \left\{ (x_1, \ldots, x_n)' \middle| \sum_{i=1}^{n} x_i = 1, \quad x_i \geq 0, \quad i = 1, \ldots, n \right\}. \quad (1)$$

An investment strategy Inv($\mathbf{x}$(t),t) is defined as a series of asset allocations over a time interval [0,T]. Portfolios are reallocated iteratively at specified rebalancing points of time, denoted by $t_j \in [0, T]$, $j = 0, \ldots, J$ and then held constant over the holding period H (in days) until the next rebalancing point of time t_{j+1}:

$$t_j \in [0,T], t_0 = 0, t_{j+1} = t_j + H, \quad j = 0, \ldots, J-1. \quad (2)$$

Asset weights are fixed at the rebalancing points of time and held constant over the holding period:

$$\forall t_j \in [0,T], j = 0, \ldots, J: \quad \mathbf{x}(t_j) = \mathbf{x}(t_j + k), k = 1, \ldots, H-1. \quad (3)$$

At the rebalancing points of time the asset weights are rebalanced in such a way that the total market value of the portfolio is not changed, as described by the following equation:

$$\langle \mathbf{x}(t_j - 1), \mathbf{y}(t_j) \rangle = \langle \mathbf{x}(t_j), \mathbf{y}(t_j) \rangle \Leftrightarrow \sum_{i=1}^{n} x_i(t_j - 1) y_i(t_j) = \sum_{i=1}^{n} x_i(t_j) y_i(t_j). \quad \forall t_j \in [0,T], \quad j = 1, \ldots, J. \quad (4)$$

where

$\mathbf{x}(t) = (x_1(t), \ldots, x_n(t))'$ the vector of asset weights at t and
$\mathbf{y}(t) = (y_1(t), \ldots, y_n(t))'$ the vector of the corresponding market prices at t.

Summarising, an investment strategy Inv($\mathbf{x}$(t),t) is described by a set of asset allocations, which satisfies the following conditions:

$$\text{(i) } \mathrm{Inv}(\mathbf{x}(t), t) := \{ \mathbf{x}(t) = (x_1(t), \ldots, x_n(t))' \quad t \in [0,T],$$
$$\text{(ii) } (x_1(t), \ldots, x_n(t))' \in I,$$
$$\text{(iii) } \mathbf{x}(t) \text{ satisfying equ. (3) and (4)} \}. \quad (5)$$

In the following the definition of an investment strategy is further specified in order to define *risk-minimising* investment strategies. In a first step, investment strategies are defined which minimise the diversifiable risk. An optimisation model is formulated, which periodically is solved, thus generating a series of diversifiable risk-minimising asset allocations. In a second step the investment strategy is embedded into a dynamic framework of portfolio insurance, thus controlling the systematic risk over the investment period.

Step 1: Generating diversifiable risk-minimising investment strategies

In the following the question is addressed as to how portfolio weights are adjusted at every rebalancing point of time $t_j \in [0, T]$, $j = 0, \ldots, J$. Different rebalancing criteria have been suggested in the literature in the context of dynamic asset allocation.[13] Often one risky asset, represented by an index, is applied, or the portfolio is reallocated according to a predefined constant mix. In this paper a modified Markowitz portfolio optimisation approach is suggested to rebalance the asset weights, in order to periodically minimise the diversifiable risk of the portfolio at every rebalancing point of time.

Traditional Markowitz portfolio optimisation approach

Modern approaches of asset allocation go back to the traditional concept of Markowitz portfolio optimisation. Markowitz suggested a mean–risk optimisation framework for investment choice and asset allocation based on the risk measure of portfolio variance. According to his framework, investors are risk averse and respond to the uncertainty of profits by selecting portfolios that yield a specified risk-return trade-off.[2] The Markowitz portfolio optimisation problem can be represented in the following form (P), where portfolio risk is minimised with respect to a specified return benchmark:

$$\text{(i)}\ \ \text{(P)}\ \text{Min}\quad \sigma^2_{PF} = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j COV_{ij}$$

$$\text{(ii)}\ \ \text{w. r. t. } \mu_{PF} = \sum_{i=1}^{n} x_i \mu_i = r_{BM},$$

$$\sum_{i=1}^{n} x_i = 1, x_i \geq 0, i = 1, \ldots, n. \quad (6)$$

where

σ^2_{PF}	the portfolio variance,
COV_{ij}	the covariance of asset i and j,
μ_{PF}	the expected portfolio return,
μ_i	the expected return of the i-th asset,
r_{BM}	the return benchmark.

The original model was built in a one-period framework. The optimal solution minimises the diversifiable risk and yields optimal portfolio diversification. The Markowitz model frequently was criticised as underlying assumptions do not hold in reality, especially with respect to the assumption of normally distributed asset returns.[3,4] The model does not address any protection against systematic market risk.

Use of new risk measures

Various extensions of the classical mean–variance framework have been suggested. Assumptions on underlying asset returns have been generalised and different asymmetric downside risk measures have been suggested for application in the mean–risk optimisation framework.[15,16] More recently the attention moved on to newer percentile characteristics such as VaR or CVaR,[4,5] which are the focus of this paper.

The α%-VaR represents the α%-quantile of the random portfolio loss $L(\mathbf{x})$, which is described by:[4]

$$VaR_{\alpha}(L(\mathbf{x})) = \inf\{z \in \Re | P(L(\mathbf{x}) \leq z) \geq \alpha\}. \quad (7)$$

While VaR measures the worst loss, which can be expected with a certain probability, it does not provide any information on extreme losses beyond VaR, which is in the focus of the risk measure CVaR. CVaR, also denoted as mean excess loss, expected shortfall, or tail VaR, yields information on the losses which are expected in the case that losses exceed VaR. For continuous distributions the α%-CVaR is defined as the conditional expectation beyond VaR:[4]

$$CVaR_{\alpha}(L(\mathbf{x})) = E[L(\mathbf{x}) | L(\mathbf{x}) \geq VaR_{\alpha}(L(\mathbf{x}))]. \quad (8)$$

This definition has been modified for discontinuous loss functions to consider jumps of the distribution function at the α-quantile.[17,18]

Recent streams of research have discussed the issue of choosing appropriate risk measures in the context of risk assessment and portfolio optimisation. Discussions affecting the choice between VaR and CVaR are based on various aspects, eg the differences of their mathematical properties, stability of statistical estimation, implementation of optimisation procedures or acceptance by regulators.[4,19–22] It was shown that CVaR has superior mathematical properties in comparison to VaR. It represents a convex function and a coherent risk measure in the sense of Artzner *et al.*,[23] as well as a coherent deviation measure in the sense of Rockafellar *et al.*[24] CVaR can be optimised and constrained by linear programming methods, whereas VaR is more difficult to optimise, as VaR is not convex and may yield different local extrema, if returns are not normally distributed.[4] On the other hand, VaR disregards the tail of the distribution beyond VaR, for which typically only poor estimates are available. Estimates based on VaR therefore often turn out to be more stable than those based on CVaR. Nevertheless CVaR provides an adequate picture of risks reflected in extreme tails, if the extreme tail losses are correctly estimated.[20] In fact, no distinct conclusion can be drawn from existing literature on the superiority of one or the other risk measure.

Modified Markowitz optimisation approach

This paper follows newer approaches of mean-risk portfolio optimisation, which generalise the underlying assumptions of the traditional Markowitz model with respect to the use of risk measure and assumptions on the underlying asset return distributions, as discussed in the previous section. While the traditional Markowitz optimisation approach uses the risk measure of portfolio variance and relies on the assumption of normal distributed asset returns, the optimisation approach followed in this paper incorporates skewness and non-normal behaviour of asset returns; and therefore also is referred to as a generalised Markowitz optimisation model.[25]

Let $\rho(\mathbf{x})$ denote a risk measure, ie a real valued function ρ: $\Re^n \rightarrow \Re$, which assigns the associated risk to the portfolio $\mathbf{x} = (x_1, \ldots, x_n)'$. Using this general definition of a risk measure and referring to the above equation (6), a modified mean-risk Markowitz optimisation can be formulated as follows:

$$\begin{aligned}&\text{(i)}\quad (P')\,\text{Min}\ \rho(\mathbf{x})\\&\text{(ii)}\quad \text{w. r. t. } \mu_{PF}=\sum_{i=1}^{n} x_i\mu_i = r_{BM},\\&\qquad \sum_{i=1}^{n} x_i = 1, x_i \geq 0, i = 1, \ldots, n.\end{aligned} \tag{9}$$

In the case study presented below, the general risk measure $\rho(\mathbf{x})$ in the objective function is specified by the risk measures VaR and CVaR on different confidence levels. Rockafellar and Uryasev have shown that, in the case where asset returns follow a normal distribution, the optimal solutions of the optimisation problem (P') are identical if the risk measure $\rho(\mathbf{x})$ is defined either in terms of VaR, CVaR or standard deviation.[4] Yet it frequently turns out that asset returns lack fitness of normal distributions, thus the use of appropriate risk measures becomes a prevailing issue of investment choice.

Definition of diversifiable risk-minimising investment strategies

A diversifiable risk-minimising investment strategy is defined by an investment strategy, which minimises the diversifiable risk periodically and solves the above optimisation problem (P') in equation (9) at the rebalancing points of time $t_j \in [0, T], j = 0, \ldots, J$. Specifying the general definition of an investment strategy in the equation (5) above, a *diversifiable risk-minimising investment strategy* $\text{Inv}_{\text{divrisk\_min}}(\mathbf{x}(t),t)$ is described as follows:

$$\begin{aligned}&\text{(i)}\quad \text{Inv}_{\text{div risk min}}(\mathbf{x}(t),t)\\&\qquad := \{\mathbf{x}(t) = (x_1(t),\ldots,x_n(t))',\\&\qquad\quad t\in[0,T],\\&\text{(ii)}\quad (x_1(t),\ldots,x_n(t))' \in I,\\&\text{(iii)}\quad (x_1(t),\ldots,x_n(t))' \text{ satisfying}\\&\qquad \text{equ. (3) and (4)},\\&\text{(iv)}\quad (x_1(t_j),\ldots,x_n(t_j))' \text{ solving}\\&\qquad (P')\\&\qquad \text{in equ. (9)}\\&\qquad \forall\, t_j \in [0,T],\\&\qquad j = 0,\ldots,J\}.\end{aligned} \tag{10}$$

Referring to line (iv) in equation (10), the investment horizon H of the investment strategy and the holding period, which is applied to estimate risk and return ratios in the periodical portfolio optimisations (P'), coincide, ie the risk and return ratios, which are applied in the portfolio optimisation at t_j, are estimated to the next rebalancing point of time t_{j+1}.

Step 2: Embedding investment strategies into dynamic portfolio insurance

Definition of portfolio insurance and risk-minimising investment strategies

In the second step portfolio insurance is applied on the diversifiable risk-minimising investment strategies, as derived from step 1 above, in order to achieve protection against systematic risk. As both the minimisation of the diversifiable risk and the control of the systematic risk are addressed, the resulting investment strategies are referred to as *risk-minimising investment strategies*. Portfolio insurance aims to maintain a specified minimum portfolio value over the investment period. Portfolio insurance thus intends to alter the return distribution in such a way that downside risk is limited, while upside potential is preserved to some extent.[26] In order to achieve protection, the portfolio value on a daily basis is compared to the floor level. If the specified floor is reached assets are reallocated to a riskless asset according to the specified insurance strategy.

Formally, the *risk-minimising investment strategy*, in the following denoted by $\text{Inv}_{\text{riskmin}}(\mathbf{x}(t),t)$, is derived from the above equation (10) by incorporating an additional asset, the cash position, into the investment universe, into which the risky portfolio is shifted, if the floor level is reached. The vector of the asset weights then is given by $\mathbf{x} = (x_1, \ldots, x_n, x_{n+1})'$, where the asset x_{n+1} represents the additional cash position and y_{n+1} the corresponding market price. A further condition is included into the strategy definition, requiring that the portfolio value is to be maintained above the floor level floor(t). Summarising, a risk-minimising investment strategy $\text{Inv}_{\text{riskmin}}(\mathbf{x}(t),t)$ is described by:

$$
\begin{aligned}
&\text{(i)} \quad \text{Inv}_{\text{risk min}}(\mathbf{x}(t), t) \\
&\qquad := \{(x_1(t), \ldots, x_n(t), x_{n+1}(t))', \\
&\qquad\quad t \in [0, T], \\
&\text{(ii)} \quad (x_1(t), \ldots, x_n(t))' \in I, \\
&\text{(iii)} \quad (x_1(t), \ldots, x_n(t))' \text{ satisfying} \\
&\qquad \text{equ. (3) and (4)}, \\
&\text{(iv)} \quad (x_1(t_j), \ldots, x_n(t_j))' \text{ solves} \\
&\qquad (P') \\
&\qquad \text{in equ. (9)} \\
&\qquad \forall\, t_j \in [0, T], \\
&\qquad j = 0, \ldots, J, \\
&\text{(v)} \quad PV(t) = \sum_{i=1}^{n+1} x_i(t) y_i(t) \\
&\qquad\qquad \geq \text{floor}(t)\ \forall\, t \in [0, T]\}.
\end{aligned} \tag{11}
$$

In the following a survey is given on different concepts of portfolio insurance. Subsequently the strategies, which are applied in the case study, are introduced.

Survey on concepts of portfolio insurance

Different insurance strategies have been suggested in the literature. The origins of portfolio insurance date back almost 40 years, when Leland and Rubinstein[27] suggested synthetic put creation. In the subsequent time a wide field of research evolved on the development of different concepts of portfolio insurance. Portfolio insurance strategies can be classified into static and dynamic strategies, depending on how often the portfolio is reallocated. Whereas static strategies require at most one reallocation, dynamic strategies require more than one allocation over time, as with eg the constant proportion portfolio insurance strategy (CPPI)[28,29] or time invariant portfolio protection (TIPP),[30] which are regaining increasing attention in practical applications and are considered in the case study.

Portfolio insurance strategies can also be distinguished by the use of instruments. Some strategies use options, as with eg the protective put strategy;

other strategies perform portfolio insurance by a systematic shift between the risky portfolio and a cash asset without the use of options, as with the stop-loss strategy and also the strategies CPPI and TIPP.

The impacts of portfolio protection have been analysed in various settings, based on historical data as well as on Monte Carlo simulations and focusing on different aspects, eg total performance, payoff patterns or risk-return ratios describing the return distribution.[13,27–31] Existing literature does not yield any clear picture on the superiority of certain portfolio insurance strategies. Rather the appropriateness of using special strategies seems to depend on specific market conditions under consideration.

Therefore, the concept of risk-minimising investment strategies leaves open which specific portfolio insurance should be applied. In the case study presented below, insurance strategies are considered, which achieve protection by shifting portfolio weights between a riskless asset (cash position) and the portfolio of risky assets. The focus is on the stop-loss strategy and on the dynamic strategies CPPI and TIPP, which are described next. Thereby the following notations are applied:

PV(t)	the portfolio value at t,
N	the initial investment,
T	the end of the investment period,
F	the protection level (‘floor’),
r_f	riskless rate.

Stop-loss strategy

The stop-loss strategy represents a simple strategy of portfolio insurance. Under stop loss, an investment in the risky asset is maintained as long as the current value of the portfolio is greater or equal to a specified protection level (‘floor’). The strategy requires liquidating the risky position if the current value PV(t) reaches the given floor. The strategy can be specified either by a constant stop-loss level or by a stop-loss level at the investment horizon, which is applied in the case study below:

(1) Constant stop-loss level:

$$PV(t) \geq F,$$

(2) Stop loss at investment horizon:

$$PV(t) \geq \exp[-r_f(T - t)]F. \quad (12)$$

The stop-loss strategy is path dependent and it does not yield any upside potential once the floor is reached, as there is no reinvestment in the risky asset. It bears gap risk, if the prices fall below the target level before the position can be sold.

Constant proportion portfolio insurance (CPPI)

The concept of CPPI goes back to work by Perold[28] and Black and Jones.[29] It comprises a set of trading rules, which systematically shift funds between a risky asset and a risk-free asset in order to guarantee the desired floor, while preserving an upside potential of a portfolio.

The CPPI is a dynamic strategy. The concept basically assumes that a total loss of the risky asset is very unlikely. The risky asset may lose some value, represented by a multiplier ‘m’ of the strategy. A given risky budget is invested m times. The portfolio value may drop by the factor 1/m within the liquidation period without missing the target level. The higher the multiplier m is chosen, the higher the risk of the investment; however, the portfolio value may drop faster in the case of a market decline. The determinants of the CPPI strategy are:

- The floor (F(t)): The floor corresponds to the minimum portfolio value, which is to be maintained over the investment period. It grows at the riskless rate. It corresponds to the discounted minimum target level F:

$$F(t) = \exp[-r_f(T - t)] \qquad (13)$$

- The cushion (C(t)): The cushion is calculated as the difference of the current portfolio value to the floor.

$$\begin{aligned} C(t) &= PV(t) - F(t) \\ &= PV(t) - \exp[-r_f(T - t)] \end{aligned} \qquad (14)$$

- The multiplier (m): The multiplier m takes on values larger than 1 and determines the risk level of the CPPI strategy.
- The exposure (E(t)): The exposure represents the portion of the portfolio allocated to the risky asset. It corresponds to the specified multiple of the cushion C(t).

$$E(t) = mC(t). \qquad (15)$$

The algorithm of the CPPI insurance is summarised in Figure 2.[13,29]

The CPPI strategy insures a floor of the portfolio performance. It is path dependent. In case the exposure E(t) drops to zero, only the minimum target level is reached. The CPPI strategy bears gap risk like the stop-loss strategy. Once the floor level is reached, CPPI remains invested in the riskless asset. It does not capture upside potential in recovering markets, and it does not protect gains above the floor level.

Time invariant portfolio protection (TIPP)

TIPP was first suggested in 1988 by Estep and Kritzman.[30] It represents a modification of the CPPI strategy. The TIPP strategy also aims to protect a preset floor. In contrast to the CPPI strategy, the floor is permanently adjusted as a specified percentage of the highest value the portfolio has reached. The parameters correspond to those of the CPPI strategy, except that the floor is defined as a percentage of the current portfolio value (F(%)).The algorithm of the TIPP strategy is summarised in Figure 3.[30]

The TIPP strategy insures a floor of performance. In upward-moving markets, the floor level is increased. In contrast to the CPPI strategy, gains are protected in this strategy. On the other hand, by raising the floor in upward-moving markets, the upside gain

Select start parameter: F, N, T, r_f, m and the rebalancing period Δt.
Step 1: Calculate current risk budget C(t) according to the above equation (14)
Step 2: Calculate exposure E(t) to the risky asset
according to the above equation (15)
Step 3:If E(t) is positive,
then invest E(t) into the risky asset and the remaining amount
in the riskless asset. Wait until time Δt has passed and go back to step 2.
If C(t) and therefore E(t) is zero,
liquidate the risky investment and invest in the riskless portfolio
until the end of the investment horizon.
Step 4: Repeat above steps until the end of the investment horizon T.

Figure 2: Algorithm of the CPPI strategy

Select start parameter: F(%), N, T, r_f, m and the rebalancing period Δt.
Step 1: Determine the portfolio value PV(t).
Step 2: Multiply the portfolio's value by the floor percentage
F(in %) * PV(t). (16)
Step 3: Define the new floor:
If the result of step 2 is greater than the previous floor,
then this result becomes the new floor, otherwise, keep the old floor.
floor(t) = max (floor(t-1), F(in %) * PV(t)). (17)
Step 4: Calculate the Cushion C(t):
C(t) = PV(t) – floor(t). (18)
Step 5: Calculate the exposure to the risky asset:
E(t) = m * C(t). (19)
Step 6: Buy or sell the risky asset until the value of the risky asset equals
the result of step 5, invest the rest, if any, riskless.
Step 7: Repeat above steps until the end of the investment horizon T.

Figure 3: Algorithm of the TIPP strategy

potential is limited, as the risky asset is decreased in an increasing market. Once the floor level is reached and the portfolio is shifted into the riskless asset, the strategy starts reinvesting into the risky portfolio by the interest earned on the riskless asset, thus allowing the portfolio value to recover from a drawdown. The TIPP strategy is also path dependent and bears gap risk.

Summary

In this section a concept of risk-minimising investment strategies was introduced. In the first step investment strategies were derived which periodically minimise the diversifiable risk, as summarised in equation (10). In the second step the investment strategies were embedded into a dynamic framework of portfolio insurance in order to achieve additional protection against the systematic market risk, as described by equation (11). Summarising, a risk-minimising investment strategy represents a series of asset allocations,

- which solve a risk-minimisation problem at all rebalancing points of time in order to minimise the diversifiable risk for each one holding period, and
- which control the systematic risk in a dynamic framework over the investment period.

In the description of risk-minimising investment strategies no certain risk measure or portfolio protection strategy was recommended for application. As discussed, no clear conclusion can be drawn from the existing literature on the general superiority of one special risk measure or a special insurance strategy. In fact, it depends on the respective market conditions and applications in view, as examined in the case study presented in the following section.

CASE STUDY ON RISK-MINIMISING INVESTMENT STRATEGIES

Survey on the case study

Objective of the case study

The case study aims to exemplify the approach of risk-minimising investment strategies, as suggested in the previous section, and to analyse the impacts of minimising diversifiable risk and controlling the systematic risk in the context of the recent financial crisis. The investment strategies are specified by the use of the downside risk measures VaR

and CVaR, and by an application of the portfolio protection strategies stop loss, CPPI and TIPP. The primary purpose of the case study is to illustrate the underlying concept, rather than simulating a real-world investment example, which is beyond the scope of this paper.
The performance of specific investment strategies and portfolio insurance strategies is analysed in a simplified portfolio setting, which is based on historical data comprising the recent financial crisis. The following questions form the focus of the case study:

- Questions with respect to *minimising diversifiable risk*:
  — Which impact does portfolio optimisation have on the performance of the investment strategies (in relation to the criterion of constant rebalancing)?
  — Does the choice of the risk measure in the optimisation have an effect on the performance of the investment strategies?
- Questions with respect to *controlling the systematic risk*:
  — How does portfolio insurance affect the performance of investment strategies?
  — Can any conclusion be drawn, which aspects have greater impact:
    (i) minimising the diversifiable risk (by portfolio optimisations) or
    (ii) controlling the undiversifiable risk (by applying portfolio insurance)?

Proceeding of the case study

The case study consists of two steps, which correspond to the proceeding of the general strategy derivation in the previous section. In step 1, investment strategies are defined and analysed, which minimise the diversifiable risk. The strategies are generated by periodical CVaR- and VaR-minimisations for the time period under consideration, comprising the recent financial crisis and a subsequent phase of market recovery. Next, three constant rebalancing investment strategies are generated. The performance of the diversifiable risk-minimising investment strategies is compared to the performance of the constant rebalancing investment strategies in order to analyse the impacts of diversifiable risk-minimisation.

In step 2 of the case study, portfolio insurance is applied on the investment strategies derived from step 1, in order to achieve protection against systematic risk. The insurance strategies of stop loss, CPPI and TIPP are considered, which have been introduced in the previous section. In order to analyse the impacts of portfolio insurance in the time period under consideration, the performance of the insured investment strategies is analysed and compared to the performance of the investment strategies without portfolio protection, as generated in step 1, thus addressing the third and fourth questions above.

In the following the strategies generated in step 1 are referred to as the 'unsecured' investment strategies, as they are not secured against systematic risk, in contrast to the 'secured' investment strategies achieved in step 2, on which portfolio insurance is applied. The term 'secured' in this context refers to protection against the systematic risk. Table 1 gives a survey on the proceeding of the case study.

Focus of the case study

The primary purpose of the case study is to illustrate the underlying conceptual

Table 1: Survey on different risk-minimising investment strategies generated in the case study

Investment strategy based on …	Step 1 Generate investment strategies	Step 2 Apply portfolio insurance on investment strategies		
99%-CVaR-risk minimisation	generate **'unsecured' strategies** (no insurance)	apply **stop-loss insurance**	apply **CPPI insurance**	apply **TIPP insurance**
99%-VaR-risk minimisation				
95%-CVaR-risk minimisation				
(80%,20%)-constant mix rebalancing				
(50%,50%)-constant mix rebalancing				
(20%,80%)-constant mix rebalancing				

methodology, and to analyse the impacts of risk minimisation and systematic risk control. Therefore a simplified portfolio setting is selected. The presented two-asset case may be extended straightforwardly to a portfolio consisting of several assets.

The case study focuses on a performance analysis in the context of the recent financial crisis, when extreme losses occurred in financial markets. Different criteria have been suggested for a performance analysis of investment strategies.[13,31] In the case study the investment strategies are analysed with respect to their performance progression over the investment period and their total profit and loss at the end of the investment period. The return distributions as well as mean, minimum and maximum returns are examined. In order to compare the level of risk of the investment strategies, risk ratios are analysed which are different from those used as risk minimisation criteria in the optimisations.

Step 1: Generating investment strategies

Objective and proceeding of step 1 of the case study

In step 1 of the case study the different 'unsecured' investment strategies are generated by applying periodical mean-risk optimisation problems, as formulated in the above equation (9). Three diversifiable risk-minimising investment strategies are generated according to equation (10) above and based on the risk measures VaR and CVaR. In order to analyse the impacts of risk minimisation, in addition, three investment strategies are generated by reallocating the portfolio assets to a constant mix on the rebalancing days.

Input data and parameter settings

The investment universe consists of two risky assets, the Dow Jones-UBS Commodity Index and the Stoxx Europe 50 Index. The set of possible asset allocations is represented by the vector $\mathbf{x} = (x_1, x_2)'$ of the two assets, where x_1 denotes the portfolio weight invested in the Dow Jones-UBS Commodity Index and x_2 the asset weight invested in the Stoxx Europe 50 Index. The set of possible asset allocations I is represented by:

$$I = \{\mathbf{x} = (x_1, x_2)' \mid x_1 + x_2 = 1,\; x_1, x_2 \geq 0\}. \tag{20}$$

The investment horizon H is defined by a period of ten trading days, ie the portfolios are re-allocated at the rebalancing points of time every ten trading days. The investment strategies are generated for the investment period [0,T], starting on 2nd January, 2008 to 31st May, 2010, thus comprising the period of the most recent financial crisis and subsequent upward market movements and consisting of 63 rebalancing points of time denoted by t_j, $j = 0, \ldots, 62$.

At every rebalancing point of time $t_j \in [0, T]$, the methodology of historical simulation is applied to forecast the return distribution to the next rebalancing point of time t_{j+1}.[32] The empirical distribution of asset prices is achieved by re-evaluating the current portfolio by the ten day-log returns observed in the historical sample. The generated sample of future prices at t_{j+1} is used as input for the mean-risk optimisation at t_j.[4]

In the case study a total sample of historical data is used, covering the time period from 2nd January, 1991 to 31st May, 2010, as illustrated in Figure 4.[33,34]

For the mean-risk optimisations on the rebalancing days t_j, $j = 1, \ldots, 62$, rolling subsets of the given total historical data sample are applied. The subset used at the first rebalancing point on 2nd January, 2008 (t_0) reaches back from 2nd January, 1991 to 31st December, 2007. The subset of the historical data then is stepwise rolled forward by ten trading day periods to each next rebalancing point of time t_j, thus each data subset, used as input for the portfolio optimisations at t_j, comprises the identical sample size of 4,395 observations.

The methodology of historical simulation has become popular in financial risk management, as it is easy to implement and does not rely on additional assumptions and parameter estimations.[35] Also it directly deals with the choice of the investment horizon, as returns are measured over intervals that correspond to the length of the horizon.[36] On the other hand, historical simulation has been criticised for technical weaknesses and little forecasting power.[35,36] As the objective of this paper is to demonstrate and analyse the underlying conceptual methodology, historical simulation is applied despite its shortcomings. Table 2 summarises the

Figure 4: Historical data used in the case study

Table 2: Parameter set-up of step 1 of the case study

Investment period	[0, T]	
Starting point	t_0	2nd January, 2008
End point	t_T	31st May, 2010
Rebalancing points	$t_j = j*H$,	$j = 0, \ldots, 62$
Holding period	H	10 trading days
Assets	x_1	DJ UBS Commodity Index (DJUBEUTR)
	x_2	Stoxx Europe 50 Index (EU0009658178)
Historical data set		2nd January,1991–31st May, 2010

Sources: Dow Jones-UBS Commodity Index; available at: www.djindices.com (accessed 20th October, 2010). Stoxx Europe 50 Index, available at: www.yahoo.com (accessed 20th October, 2010).

parameter settings and required input data for step 1 of the case study.

Set-up of the optimisation problems

In order to generate the investment strategies based on portfolio optimisations, at every rebalancing point of time mean-risk optimisations are conducted. Asset weights are adjusted according to the optimal solutions of the portfolio optimisation problems and then kept constant over the holding period until the next rebalancing day.

In the following the set-up and the specification of the general optimisation model from equation (9) are described. At first the definition of a return benchmark r_{BM} is considered. As the minimum of expected asset returns of samples before the financial crisis exceeded the maximum expected asset returns of samples comprising periods of the market crisis, no common return benchmark can be determined from the empirical distributions which yields feasible solutions for all optimisation problems. Moreover, an identical return benchmark r_{BM} used in all optimisations (P′) over the investment period would refer to solutions of different levels of risk. Therefore, the specification of a common constant return benchmark is abandoned. Instead, at every rebalancing point of time the overall risk minimum portfolio is identified. In this way, a homonymous optimisation criterion is applied to all optimisations over the investment period.

Before specifying the risk measures applied in the objective function of the optimisation problem, the empirical distributions of the underlying asset returns are analysed. A test of goodness of fit is conducted to analyse if the asset returns are normally distributed. According to Rockafellar and Uryasev, the optimal solutions of the optimisation problem using risk measures VaR, CVaR or standard deviation coincide if asset returns follow a normal distribution.[4] Based on a X^2-test for the test of normality of asset returns, the hypothesis of normal distribution of the asset returns is rejected at the 99.9 per cent confidence level for both assets. For additional illustration, Figure 5 shows the frequency distribution of the asset returns. The black graph depicts the corresponding normal distribution.

As the asset returns are not normally distributed, different risk measures are applied in the optimisation models in order to analyse their impact on diversifiable risk minimisation. VaR and

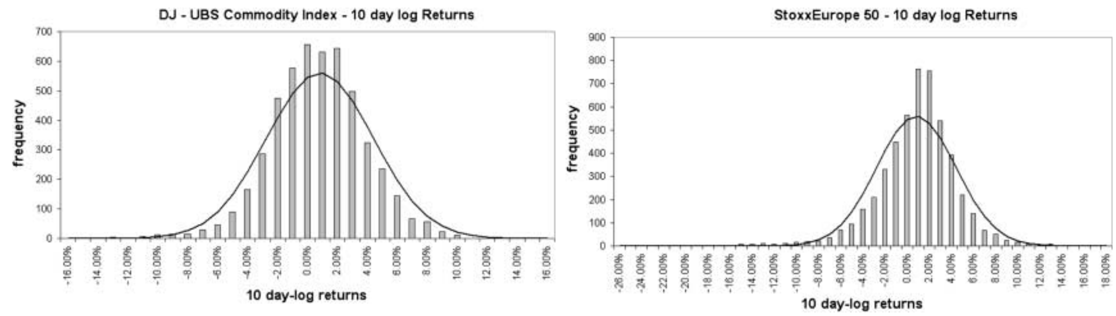

Figure 5: Frequency return distributions of underlying assets

CVaR are defined as deviation measures in the sense of Rockafellar and Uryasev.[24] The loss function L in this case is defined as a negative deviation from an expected outcome: $L(\mathbf{x}, \mathbf{y}) := E[\mathbf{y}]'\mathbf{x}\text{-}\mathbf{y}'\mathbf{x}$.

In the case study the confidence level is specified by $\alpha = 99\%$. Yet, VaR and CVaR are difficult to compare even at the same confidence level, as they are representing different levels of risk. Indeed, for a specific portfolio $\mathbf{x}$ it is possible to determine a confidence level $\alpha^\star$, such that α-VaR($\mathbf{x}$) $\approx$ $\alpha^\star$-CVaR($\mathbf{x}$). Nevertheless the choice of $\alpha^\star$ is dependent on the specific composition of the portfolio $\mathbf{x}$ and cannot be applied as an *ex ante* optimisation criterion. In the case study therefore the 99%-VaR is compared to CVaR in terms of an upper and lower bound. By definition, the 99%-CVaR represents an upper bound of the 99%-VaR. By an examination of the empirical return distributions generated at all rebalancing points of time, the 95%-CVaR of all portfolios under consideration was lower than the 99%-VaR and therefore is applied in the case study as the lower bound of the 99%-VaR.

Summarising, the following problem specifications (P_1), (P_2) and (P_3) are derived from the general optimisation problem (P') in equation (9) above, which are applied in the case study to generate the different diversifiable risk-minimising investment strategies:

$$\begin{aligned}
&(P_1)\ \text{Minimisation of 99\%-CVaR:}\\
&(i)\quad \text{Min CVaR}_{99\%}((x_1, x_2))\\
&(ii)\quad \text{w. r. t. } x_1 + x_2 = 1, x_1, x_2 \geq 0.\\
&(P_2)\ \text{Minimisation of 99\%-VaR:}\\
&(i)\quad \text{Min VaR}_{99\%}((x_1, x_2))\\
&(ii)\quad \text{w. r. t. } x_1 + x_2 = 1, x_1, x_2 \geq 0.\\
&(P_3)\ \text{Minimisation of 95\%-CVaR:}\\
&(i)\quad \text{Min CVaR}_{95\%}((x_1, x_2))\\
&(ii)\quad \text{w. r. t. } x_1 + x_2 = 1, x_1, x_2 \geq 0.
\end{aligned} \tag{21}$$

The CVaR-optimisation problems (P_1) and (P_3) are solved by an application of the Rockafellar and Uryasev optimisation algorithm.[4] The algorithm uses a sample of price scenarios as input for the risk estimation. As described in the previous section, at every rebalancing point of time t_j therefore an empirical distribution of future returns at t_{j+1} is generated by a historical simulation on the corresponding subset of the data sample. The VaR-optimisations in the problem (P2) are solved by quantile minimisations on the given empirical data set. (Different starting points are applied, as the VaR-optimisations may yield different local minima owing to the non-convexity of the VaR-function. The solutions of the VaR-optimisations based on different starting points converged to the solutions presented.)

Generating the investment strategies

By applying the optimisation problems as summarised in the above equation (21), three diversifiable risk-minimising investment strategies are generated. At every rebalancing point of time the risk minimisation problems (P1), (P2) and (P3) are solved. By specifying the above definition in equation (10), the following diversifiable risk-minimising strategies are applied in the case study:

(1) Definition of the 99%-CVaR-minimising investment strategy:

$$\begin{aligned} &\mathrm{Inv}_{99\%-\mathrm{CVaR\,min}}(\mathbf{x}(t),t)\\ &\quad=\{\mathbf{x}(t)=(x_1(t),x_2(t))'\in I,\\ &\quad\mathbf{x}(t)\ \text{satisfying equ. (3) and (4)}\\ &\quad\mathbf{x}(t_j)\ \text{solving (P1) in equ. (21)}\\ &\quad\forall\ t_j,\\ &\quad j=0,\ldots,62\}. \end{aligned}$$

(2) Definition of the 99%-VaR-minimising investment strategy:

$$\begin{aligned} &\mathrm{Inv}_{99\%-\mathrm{VaR\,min}}(\mathbf{x}(t),t)\\ &\quad=\{\mathbf{x}(t)=(x_1(t),x_2(t))'\in I,\\ &\quad\mathbf{x}(t)\\ &\quad\text{satisfying equ. (3) and (4)},\\ &\quad\mathbf{x}(t_j)\ \text{solving (P2) in equ. (21)}\\ &\quad\forall\ t_j,\\ &\quad j=0,\ldots,62\}. \end{aligned}$$

(3) Definition of the 95%-CVaR-minimising investment strategy:

$$\begin{aligned} &\mathrm{Inv}_{95\%-\mathrm{CVaR\,min}}(\mathbf{x}(t),t)\\ &\quad=\{\mathbf{x}(t)=(x_1(t),x_2(t))'\in I,\\ &\quad\mathbf{x}(t)\ \text{satisfying equ. (3) and (4)},\\ &\quad\mathbf{x}(t_j)\ \text{solving (P3) in equ. (21)}\\ &\quad\forall\ t_j,\\ &\quad j=0,\ldots,62\}. \end{aligned} \tag{22}$$

In order to analyse the impact of the risk minimisations, as addressed by the first listed question above, for comparison, investment strategies are generated which rebalance the portfolios to constant weights. A constant mix of (80%, 20%), (50%, 50%) and (20%, 80%) thereby is considered, resulting in the following investment strategies:

a) Definition of the (80%, 20%)-constant rebalancing investment strategy:

$$\begin{aligned} &\mathrm{Inv}_{(80\%,20\%)\mathrm{const}}(\mathbf{x}(t),t)\\ &\quad=\{\mathbf{x}(t)=(x_1(t),x_2(t))'\in I,\\ &\quad\mathbf{x}(t)\ \text{satisfying equ. (3) and (4)},\\ &\quad\mathbf{x}(t_j)=(80\%,20\%)'\\ &\quad\forall\ t_j,\\ &\quad j=0,\ldots,62\}. \end{aligned}$$

b) Definition of the (50%, 50%)-constant rebalancing investment strategy:

$$\begin{aligned} &\mathrm{Inv}_{(50\%,50\%)\mathrm{const}}(\mathbf{x}(t),t)\\ &\quad=\{\mathbf{x}(t)=(x_1(t),x_2(t))'\in I,\\ &\quad\mathbf{x}(t)\ \text{satisfying equ. (3) and (4)}, \end{aligned}$$

$$\mathbf{x}(t_j) = (50\%, 50\%)' \\ \forall\, t_j, \\ j = 0, \ldots, 62\}.$$

c) Definition of the (20%, 80%)-constant rebalancing investment strategy:

$$\mathrm{Inv}_{(20\%,80\%)\mathrm{const}}(\mathbf{x}(t), t) \\ = \{\mathbf{x}(t) = (x_1(t), x_2(t))' \in I, \\ \mathbf{x}(t)\ \text{satisfying equ. (3) and (4)}, \\ \mathbf{x}(t_j) = (20\%, 80\%)' \\ \forall\, t_j, \\ j = 0, \ldots, 62\}. \tag{23}$$

For all investment strategies derived according to equations (22) and (23) above, the daily performance PV(t) is calculated over the time interval [0,T] on a daily basis by multiplying the portfolio weights by the daily prices of the corresponding indices:

$$PV(t) = x_1(t)y_1(t) + x_2(t)y_2(t), \\ t = 0, \ldots, T. \tag{24}$$

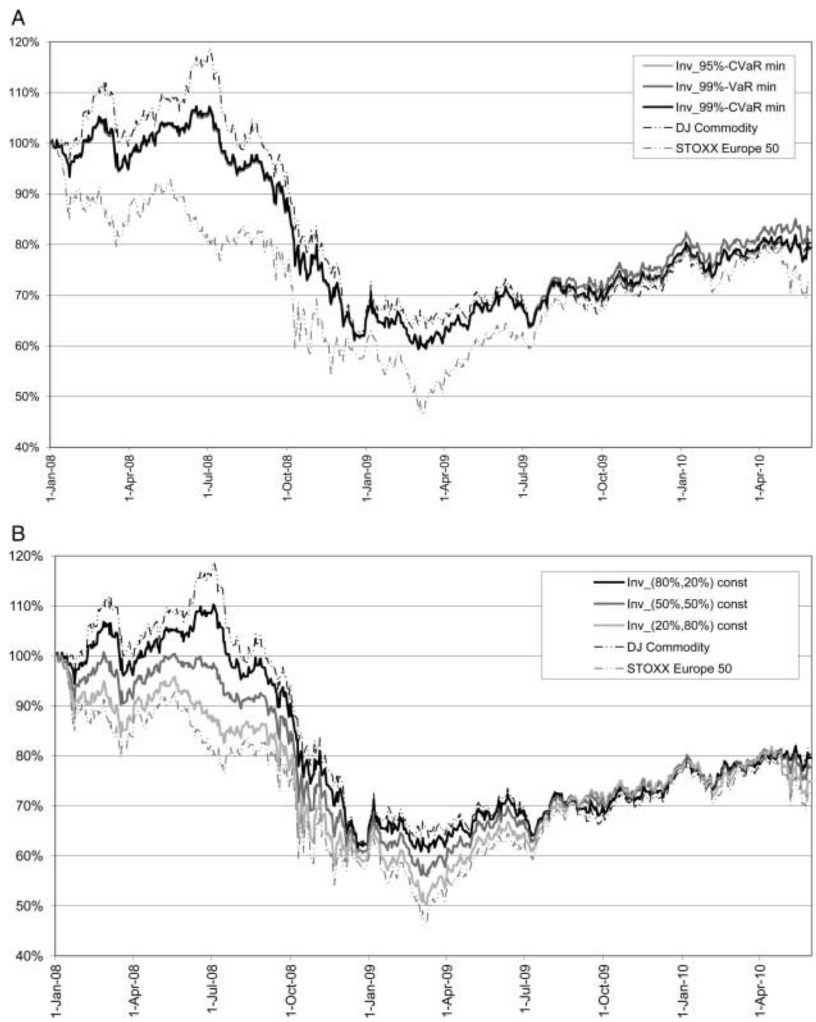

Figure 6: (a) Performance of the unsecured diversifiable risk-minimising investment strategies; (b) performance of the unsecured constant rebalancing investment strategies

Observations on Step 1 of the case study

The performance PV(t) of the different investment strategies is illustrated in the following figures. Figure 6a summarises the performance of the diversifiable risk-minimising strategies, as described in the above equation (22). For comparison, the performance of the two underlying single assets is also included using dotted lines.

The diversifiable risk-minimising investment strategies yield almost identical performance before and during the period of financial crisis, regardless of whether the 99%-VaR, 99%-CVaR or 95%-CVaR are applied as risk measures. This seems remarkable, as by definition CVaR captures extreme losses in a better way than VaR. Furthermore, in the recovering market in the second half of 2009, the 99%-VaR-minimising strategy yields considerably higher performance in comparison to both CVaR strategies of identical and lower confidence levels. This result seems to some extent surprising. One might rather have expected that the 99%-CVaR-minimising strategy, by definition allowing for higher risk than the 99%-VaR-minimising strategy, might yield a higher positive performance in an upward moving market.

Figure 6b illustrates the results for the constant rebalancing investment strategies, as described in equation (23) above. For comparison, the performance of the two underlying single assets is included using dotted lines.

The constant mix strategies show a greater variation of performance. In the sideward moving markets of the first half of 2008 the (80%, 20%)-constant mix investment strategy yields high performance, which is even superior to the VaR- and CVaR-risk-minimising strategies. The (50%, 50%)- and (20%, 80%)-constant mix investment strategies yield significantly lower performance. All strategies face severe losses in the period of market downturn in the second half of 2008 and beginning of 2009.

Analysing the total performance of all investment strategies at the end of the investment period at 31st May, 2010, which is summarised in Table 3, the 99%-VaR-minimising strategy proves to be the most stable, yielding a total performance of −17.13 per cent, followed by the (80%, 20%)-rebalancing strategy (−20.35 per cent) and the 99%-CVaR-minimising strategy (−20.68 per cent). The other constant balancing strategies yield considerably lower performance than the risk-minimising strategies. The performance of the underlying assets is also included.

The return distributions of the diversifiable risk-minimising investment strategies bear lower risk and yield higher return ratios in comparison to the constant rebalancing strategies, as summarised in Table 9 in Annex 2. The risk of the investment strategies is analysed by a CVaR-estimation at the lower confidence levels of 90% and 93%. Remarkably, the 99%-VaR-minimising strategy yields the lowest 93%- and 90%-CVaRs. Mean returns of all strategies over the investment period are negative. The optimised strategies yield slightly more stable mean returns than the (50%, 50%)- and the (20%, 80%)-rebalancing strategy. The VaR-minimising strategy also proves the most stable in terms of maximum, minimum and mean returns.

Summary of findings for step 1 of the case study

The diversifiable risk-minimising investment strategies tend to yield more

Table 3: Total performance of the unsecured investment strategies

Investment strategies	Portfolio value PV(t = 0)	Portfolio value PV(t = T)	Total profit and loss
$Inv_{99\%\text{-CVaR min}}$	100.00	79.32	−20.68%
$Inv_{99\%\text{-VaR min}}$	100.00	82.87	−17.13%
$Inv_{95\%\text{-CVaR min}}$	100.00	79.00	−21.00%
$Inv_{(80\%,20\%)\text{ const}}$	100.00	79.65	−20.35%
$Inv_{(50\%,50\%)\text{ const}}$	100.00	77.79	−22.21%
$Inv_{(20\%,80\%)\text{ const}}$	100.00	74.95	−25.05%
DJ Commodity	100.00	80.32	−19.68%
Stoxx Europe 50	100.00	72.58	−27.42%

stable performance than the constant rebalancing investment strategies, independently of the direction of market movements. Portfolio optimisation has a stabilising impact on the performance of the investment strategies. Focusing on the period of market crisis in the second half of 2008 and beginning of 2009, all investment strategies, regardless of whether they are optimised or constantly balanced, face serious losses, from which they only recover to a lesser extent until the end of the investment period at 31st May, 2010.

The example also illustrates that a constantly rebalancing investment strategy might yield a better performance by chance. Yet it is not known in advance *which* constant mix strategy will be superior and should be chosen *ex ante*. Choosing the right constant mix remains a game of hazard from this point of view. In this case it might have been unlikely that an investor would have invested into a long-term asset allocation of 80% commodities and 20% equities investment.

Remarkably, in the context of the case study, the choice of the downside risk measure in the optimisations does not have any significant impact on the performance of the investment strategies in the period of market decline. By its technical definition CVaR has methodological advantages and captures tail risk in a better way than VaR. Yet it does not show its strengths in the example under consideration. In the recovering market, the VaR-minimising strategy yields a superior performance with respect to both CVaR-minimising strategies.

Step 1 of the case study illustrates that portfolio optimisation has a positive impact on the performance of the investment strategies independently of the direction of market movements. However, it also exemplifies that portfolio optimisation does not protect against serious losses in times of systematic market crises.

Step 2: Applying portfolio insurance

Objective and proceeding of step 2 of the case study

In step 2 of the case study additional portfolio insurance is applied on the investment strategies derived from step 1. The impact of reducing the systematic market is analysed, as addressed by the third and fourth listed questions above. The portfolio insurance by stop loss, CPPI and TIPP is applied.

Parameter setting of step 2 of the case study

The following parameters are chosen for step 2 of the case study. With respect to the CPPI and the TIPP strategies, the strategy multiplier is specified by m = 5. This corresponds to a typical setting in practical applications and also ensures that the CPPI and the TIPP strategies are fully invested in the risky portfolio at the beginning of the investment period. Portfolio insurance is conducted on a daily basis. Further settings are applied as summarised in Table 4.

As the case study aims to illustrate the underlying methodology rather than simulating a real-world investment example, further strategy specifications of practical applications, eg transaction costs, trading filters or others, are not considered in the case study.

Table 4: Parameter set-up for step 2

Strategy start	t = 0	2nd January, 2008
Strategy end	t = T	31st May, 2010
Strategy multiplier	m	5
Investment amount	N	100
Floor percentage	F(%)	80%
Floor	F	80
Risk-free rate of return	r_f	1.50%

Observations on step 2 of the case study

Observations of applying stop-loss insurance

The following figures summarise the performance of all investment strategies under the stop-loss insurance and may be compared to the performance of the unsecured strategies in Figures 6a and 6b above. Figure 7a displays the results for diversifiable risk-minimising investment strategies, Figure 7b for the constant rebalancing investment strategies.

The figures illustrate that the application of stop-loss insurance results in an identical total performance of all investment strategies at the end of the investment period, as in October 2008 all strategies hit the floor level. The risky portfolio is shifted into the riskless asset in all cases, yielding an identical total loss of −20 per cent. This results in more stable results with respect to the unsecured case for all strategies except the 99%-VaR-minimising strategy. The upside potential in recovering markets is not captured by the stop-loss strategy. Figure 7a also highlights that the use of different risk measures in the optimisations does not have a significant impact on the performance of the investment strategies; as the performance graphs show almost identical progression. The detailed progression of the stop-loss strategy for the 99%-CVaR-minimising and the 99%-VaR-minimising investment strategies is illustrated in Figures 16 and 17 in Annex 3. Table 5 summarises the total performance of the stop-loss insured investment strategies.

Comparing the return frequency distributions of the unsecured and the stop-loss insured investment strategies, a reduction of the tails of the distribution under stop loss can be observed, as illustrated in Figures 12 and 13 in Annex 1. Under stop loss, maximum losses as well as maximum returns are reduced, as summarised in Table 10 of Annex 2.

Observations of applying CPPI insurance

The following figures illustrate the performance of CPPI insured risk-minimising strategies (Figure 8a) and

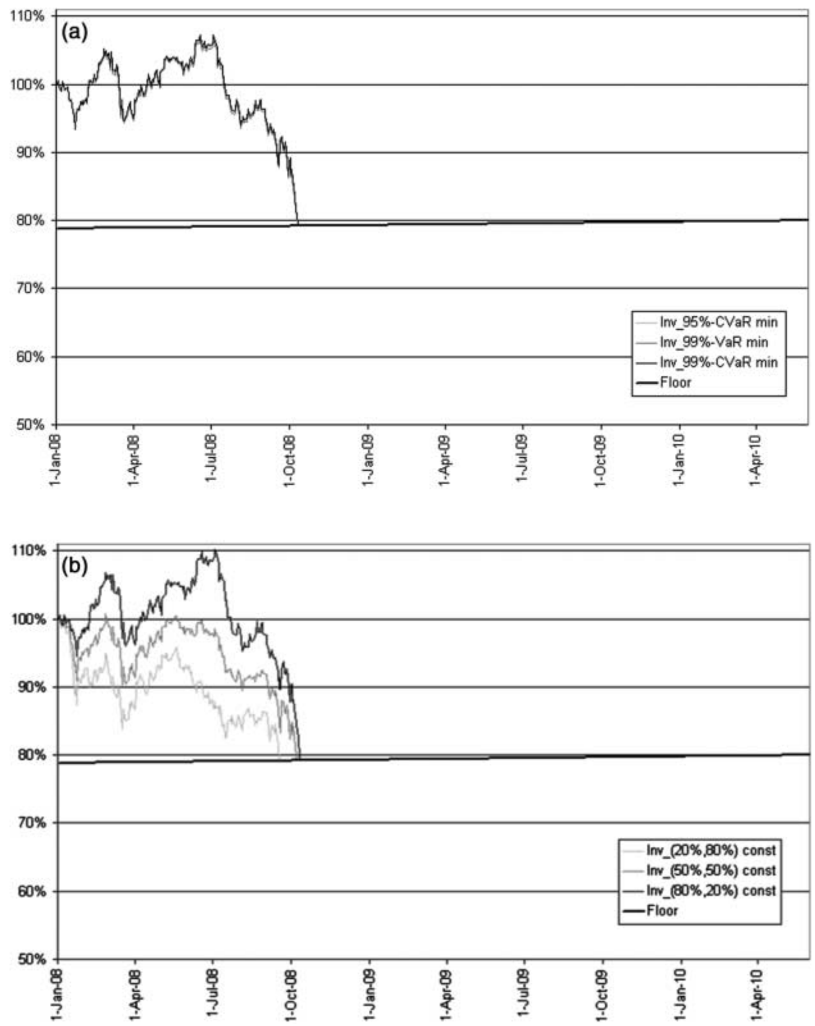

Figure 7: (a) Performance of the risk-minimising investment strategies under stop-loss insurance; (b) performance of the constant rebalancing investment strategies under stop-loss insurance

constant mix rebalancing strategies, the (Figure 8b). The charts illustrate that the application of CPPI insurance yields an identical total performance at the end of the investment horizon for all investment strategies, as all strategies hit the floor level in October 2008 under the CPPI protection as well. In comparison to the stop loss and the unsecured investment strategies, the performance under CPPI is lower in the first half of 2008, as cash positions are built up in this period. The upside potential of market recovery in the first months of 2010 is not captured by the CPPI protection either. The detailed progression of the CPPI insurance of the 99%-CVaR- and 99%-VaR-minimising investment strategies is illustrated in Figures 14 and 15 in Annex 3.

Analysing the risk and return ratios of the resulting return distributions, as summarised in Table 11 in Annex 2, the risk measures 90%- and 93%-CVaR are further reduced under CPPI, and mean returns are slightly increased with respect to the unsecured strategies and the stop-loss insured strategies. Table 6 summarises the total performance

Table 5: Total performance under stop-loss insurance

Stop loss	Portfolio value PV(t = 0)	Portfolio value PV(t = T)	Total performance	Hit floor level
$Inv_{99\%\text{-CVaR min}}$	100.00	80.00	−20.00%	10 Oct / 2008
$Inv_{99\%\text{-VaR min}}$	100.00	80.00	−20.00%	10 Oct / 2008
$Inv_{95\%\text{-CVaR min}}$	100.00	80.00	−20.00%	10 Oct / 2008
$Inv_{(80\%,20\%)\text{ const}}$	100.00	80.00	−20.00%	10 Oct / 2008
$Inv_{(50\%,50\%)\text{ const}}$	100.00	80.00	−20.00%	08 Oct / 2008
$Inv_{(20\%,80\%)\text{ const}}$	100.00	80.00	−20.00%	16 Sept / 2008

results of an application of CPPI insurance on the investment strategies for the risk-minimising and the constant rebalancing investment strategies.

Observations of applying time invariant portfolio protection (TIPP)

The following figures illustrate the performance of the TIPP insured

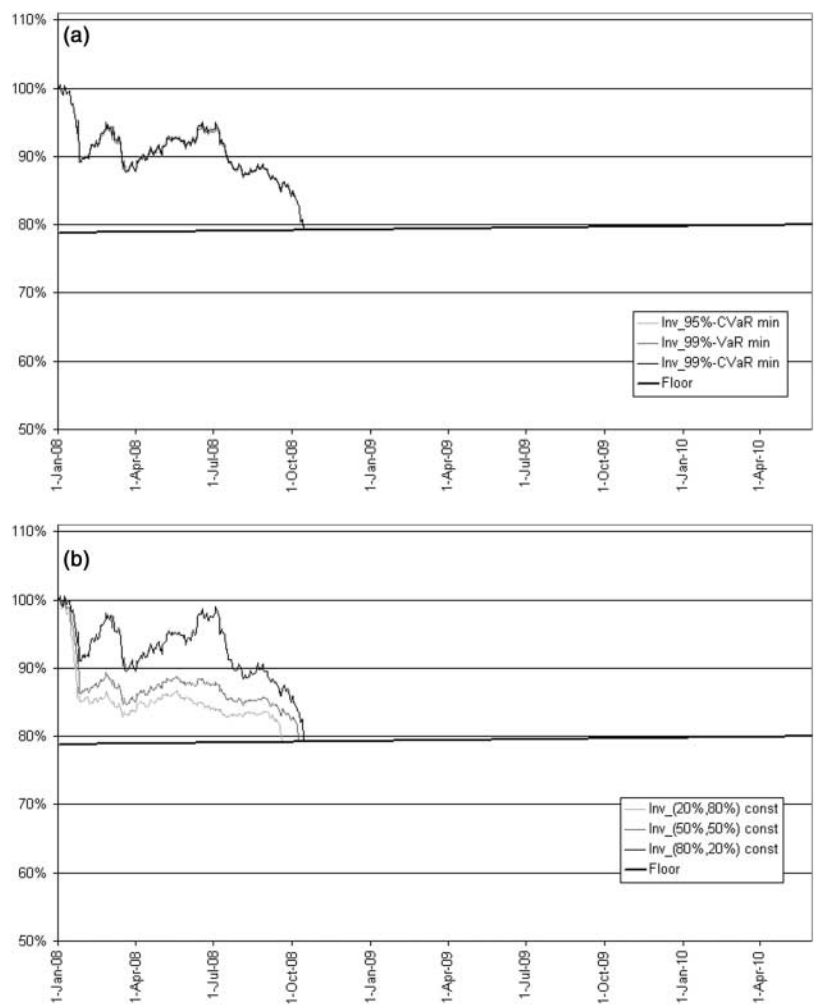

Figure 8: (a) Performance of the risk-minimising investment strategies under CPPI; (b) performance of the constant rebalancing investment strategies under CPPI

Table 6: Total performance under CPPI

CPPI	Portfolio value PV(t = 0)	Portfolio value PV(t = T)	Total performance	Hit floor level
$Inv_{99\%\text{-CVaR min}}$	100.00	80.00	−20.00%	14 Oct / 2008
$Inv_{99\%\text{-VaR min}}$	100.00	80.00	−20.00%	14 Oct / 2008
$Inv_{95\%\text{-CVaR min}}$	100.00	80.00	−20.00%	14 Oct / 2008
$Inv_{(80\%,20\%)\text{ const}}$	100.00	80.00	−20.00%	14 Oct / 2008
$Inv_{(50\%,50\%)\text{ const}}$	100.00	80.00	−20.00%	08 Oct / 2008
$Inv_{(20\%,80\%)\text{ const}}$	100.00	80.00	−20.00%	16 Sept / 2008

risk-minimising strategies (Figure 9a) and constant mix rebalancing strategies (Figure 9b).

In comparison to stop loss and CPPI insurance it can be observed that the performance of the investment strategies is significantly improved under an application of TIPP insurance. The initial floor level is maintained. The risk-minimising investment strategies almost

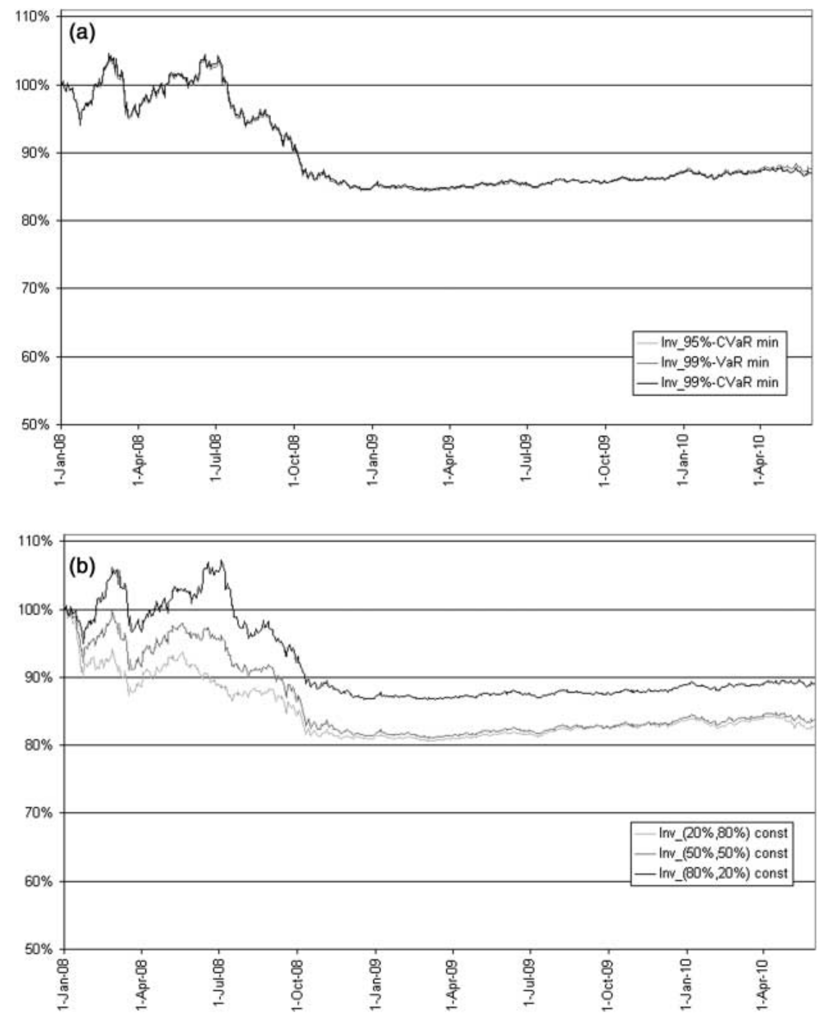

Figure 9: (a) Performance of the risk-minimising investment strategies under TIPP; (b) performance of the constant rebalancing investment strategies under TIPP

yield identical performance under the TIPP insurance. For the constant rebalancing strategies, the (80%, 20%)-strategy yields the best performance.

In the second half of 2008 the risky position of all investment strategies is rapidly decreased and the corresponding cash position increased. All investment strategies hit the floor level under TIPP in December 2008, thus protecting the portfolios against further market downturns. The strategies start reinvesting into the risky asset in the beginning of 2009, resulting in a significantly more stable performance with respect to the unsecured and the stop-loss and CPPI secured investment strategies.

The total loss clearly is reduced in comparison to the other insurance strategies and the unsecured investment strategies. The (80%, 20%)-rebalancing strategy yields the most stable total result under TIPP insurance, followed by the 99%-VaR-minimising strategy and both CVaR-minimising strategies, as is summarised in Table 7.

Analysing the return ratios, as summarised in Table 12 of Annex 2, it can be observed that maximum, mean and minimum returns are higher under TIPP in comparison to the other insurance strategies. The risk measures 90%- and 93%-CVaR are significantly reduced under TIPP for all investment strategies as well.

Summary of findings for step 2 of the case study

Table 8 summarises the total performance of all investment strategies at the end of the investment period at 31st May, 2010. The different investment strategies derived from step 2 by application of portfolio insurance are compared to the unsecured strategies of step 1. The table highlights that in the period of market decline under consideration the total performance is stabilised by applying portfolio insurance. Yet, upside potential is not captured in all cases, as for the VaR-minimising strategy, the total performance of the unsecured VaR-strategy is higher than under stop loss and CPPI insurance.

The risk-minimising strategies yield similar performance under the additional portfolio insurance. The choice of risk measure in the risk minimisations does not show any material impact on the performance of the investment strategies under portfolio insurance.

The application of stop loss and CPPI protection secures the predefined floor level in the period of market downturns, without capturing any upside potential in the recovering markets. The application of TIPP insurance yields an improved performance above the predefined floor level, as the floor level is increased before the market crisis and upside potential is captured by a reinvestment in the risky asset after the market downturn.

The results for step 2 of the case study highlight that, in times of unstable market conditions, when market declines are to be expected, portfolio insurance should be applied in order to protect portfolios against severe losses. In upward-moving markets, however, the performance of insured strategies tends to be lower, as higher cash positions are maintained. In the example under consideration, the TIPP strategy yielded best performance results, as in addition to the downside

Table 7: Total performance under TIPP

TIPP	Portfolio value PV(t = 0)	Portfolio value PV(t = T)	Total performance
$Inv_{99\%\text{-CVaR min}}$	100.00	87.11	−12.89%
$Inv_{99\%\text{-VaR min}}$	100.00	87.73	−12.27%
$Inv_{95\%\text{-CVaR min}}$	100.00	87.07	−12.93%
$Inv_{(80\%,20\%)\text{ const}}$	100.00	89.10	−10.90%
$Inv_{(50\%,50\%)\text{ const}}$	100.00	83.71	−16.29%
$Inv_{(20\%,80\%)\text{ const}}$	100.00	82.78	−17.22%

Table 8: Comparison of the total performance of all investment strategies

Investment strategies	Unsecured (step 1)	Stop loss (step 2)	CPPI (step 2)	TIPP (step 2)
$Inv_{99\%\text{-CVaR min}}$	−20.68%	−20.00%	−20.00%	−12.89%
$Inv_{99\%\text{-VaR min}}$	−17.13%	−20.00%	−20.00%	−12.27%
$Inv_{95\%\text{-CVaR min}}$	−21.00%	−20.00%	−20.00%	−12.93%
$Inv_{(80\%,20\%)\text{ const}}$	−20.35%	−20.00%	−20.00%	−10.90%
$Inv_{(50\%,50\%)\text{ const}}$	−22.21%	−20.00%	−20.00%	−16.29%
$Inv_{(20\%,80\%)\text{ const}}$	−25.05%	−20.00%	−20.00%	−17.22%

protection the strategy also captured upside potential and locked in gains. Nonetheless, the assessment of future market movements and the decision on the application of portfolio insurance is left to the investor.

Figures 10 and 11 summarise the application of the portfolio insurance strategies on the 99%-CVaR- and the 99%-VaR-minimising investment strategies. The charts highlight that the best performance is achieved under the TIPP insurance. The charts also illustrate that the downside protection is achieved at the cost of upside potential.

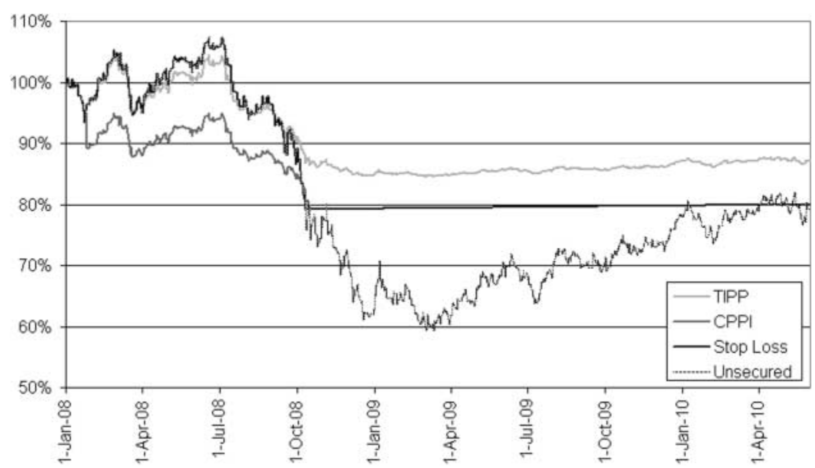

Figure 10: Performance of the 99%-CVaR-minimising investment strategies

Figure 11: Performance of the 99%-VaR-minimising investment strategies

SUMMARY AND CONCLUSION

In this paper an approach for risk–minimising investment strategies was discussed, which combines concepts of portfolio optimisation and portfolio insurance. Investment strategies were generated by embedding revolving one-period optimisations into a framework of dynamic asset allocation, which minimises the diversifiable risk and controls the systematic market risk over the investment period. A case study was presented to demonstrate the suggested methodology and to analyse the impacts of minimising diversifiable risk and controlling the systematic risk using the focus of the recent financial crisis. With respect to the portfolio optimisations, the newer risk measures VaR and CVaR were used in the risk minimisations, and the stop loss, CPPI and TIPP strategies were applied for portfolio insurance.

The case study demonstrated that, independently of the direction of market movements, portfolio optimisation improves the performance of investment strategies. Remarkably, the choice of risk measure in the optimisations did not have any significant impact on the performance of the investment strategies in the period of market decline. The example also illustrated that a constant mix rebalancing investment strategy might yield better performance results by chance.

The case study also highlighted that portfolio optimisation does not protect portfolios against severe losses owing to systematic market movements. In the period of the financial crisis under consideration all portfolios, regardless of whether constantly rebalanced or optimised, faced drastic downturns. By the application of portfolio insurance, the performance of all investment strategies was stabilised. Target floor levels were maintained, although at the cost of lower upside potential. The TIPP strategy yielded the best performance results, as in addition to the downside protection the strategy also captured upside potential and locked in gains.

In the period of market decline under consideration, the impacts of portfolio insurance overlaid the effects of portfolio optimisation on the total performance. The results of the case study give reason to suggest that, in times of financial instability, when market crises are to be expected, portfolios primarily should be protected against downside risk by adequate portfolio insurance. Yet, portfolio insurance is achieved at the cost of lower upside potential in an increasing number of markets. The application of portfolio insurance remains the investor's decision, depending on his assessment of the market trends.

Further research is needed to prove the observations of the case study in a more general context. The primary purpose of the case study was to demonstrate and analyse the suggested conceptual approach by a simplified example. It may be extended to a more realistic application by taking into consideration more complex portfolios and further aspects of a practical simulation, such as transaction costs, trading filters or others. More sophisticated statistical methodologies of risk and return estimation may be applied and observations may be further generalised, eg by examining different market conditions, varying the input parameters or applying Monte Carlo simulation instead of using historical data.

ACKNOWLEDGMENTS

The author wishes to thank Dr Dominik Dersch, Beratung, Munich, Germany, for his valuable suggestions and input on this work and Dr Stefan Schulz, Cross Asset Research, Corporate and Investment Banking, UniCredit Bank AG, Munich, Germany, for his support and providing access to the Cross Asset Optimiser, which was used for computational verifications.[37]

References

1 'Investment strategy', available at: http://en.wikipedia.org/wiki/Investment_strategy (accessed 15th October, 2010).
2 Markowitz, H. (1952) 'Portfolio selection', *The Journal of Finance*, Vol. 7, No. 1, pp. 77–91.
3 Michaud, R. and Michaud, R. (eds) (2008) 'Efficient Asset Management — A Practical Guide to Stock Portfolio Optimization and Asset Allocation', Oxford University Press, New York, NY.
4 Rockafellar, R. T. and Uryasev, S. (2000) 'Optimization of conditional value-at-risk', *Journal of Risk*, Vol. 2, No. 3, pp. 21–41.
5 Bertsimas, D., Lauprete, G. J. and Samarov, A. (2004) 'Shortfall as a risk measure: Properties, optimization, and applications', *Journal of Economic Dynamics & Control*, Vol. 28, No. 7, pp. 1353–1381.
6 Chellathurai, T. and Draviam, T. (2008) 'Markowitz principles for multi-period portfolio selection problems with moments of any order', *Proceedings of the Royal Society A*, Vol. 464, No. 2092, pp. 827–854.
7 Merton, R. C. (1969) 'Lifetime portfolio selection under uncertainty: The continuous-time case', *Review of Economics and Statistics*, Vol. 51, No. 3, pp. 247–257.
8 Hakansson, N. (1969) 'Optimal investment and consumption strategies under risk, an uncertain lifetime, and insurance', *International Economic Review*, Vol. 10, No. 3, pp. 443–466.
9 Mulvey, J. M. and Ziemba, W. T. (eds) (1998) 'Worldwide Asset and Liability

Modeling', Cambridge University Press, Cambridge.
10 Dempster, M. A. H. (ed.) (2002) 'Risk Management: Value at Risk and Beyond', Cambridge University Press, Cambridge.
11 Emmer, S., Klüppelberg, C. and Korn, R. (2001) 'Optimal portfolios with bounded capital-at-risk', *Mathematical Finance*, Vol. 11, No. 4, pp. 365–384.
12 Sun, W., Fan, A., Chen, L.-W. *et al.* (2006) 'Optimal rebalancing for institutional portfolios', *Journal of Portfolio Management*, Vol. 32, No. 2, pp. 33–43.
13 Dersch, D. (2008) 'Portfolio insurance in rough seas', Quantitative Cross Asset Research, Quantitative Cross Asset Special, December, pp. 1–24.
14 Annaert, J., Van Osselaer, S. and Verstraete, B. (2009) 'Performance evaluation of portfolio insurance strategies using stochastic dominance criteria', *Journal of Banking and Finance*, Vol. 33, No. 2, pp. 272–280.
15 Konno, H. and Shirakawa, H. (1994) 'Equilibrium relations in the mean-absolute deviation capital market', *Asia-Pacific Financial Markets*, Vol. 1, No. 1, pp. 21–35.
16 Bawa, V. S. and Lindenberg, E. B. (1977) 'Capital market equilibrium in a mean-lower partial moment framework', *Journal of Financial Economics*, Vol. 5, No. 2, pp. 189–200.
17 Rockafellar, R. T. and Uryasev, S. (2002) 'Conditional value-at-risk for general loss distributions', *Journal of Banking and Finance*, Vol. 26, No. 7, pp. 1443–1471.
18 Acerbi, C. and Tasche, D. (2002) 'On the coherence of expected shortfall', *Journal of Banking and Finance*, Vol. 26, No. 7, pp. 1487–1503.
19 Acerbi, C. and Tasche, D. (2002) 'Expected shortfall: A natural coherent alternative to value at risk', *Economic Notes*, Vol. 31, No. 2, pp. 379–388.
20 Sarykalin, S., Serraino, G. and Uryasev, S. (2008) 'Value-at-Risk vs. conditional value-at-risk in risk management and optimization', *Tutorials in Operations Research*, Informs 2008, pp. 270–294.
21 Yamai, Y. and Yoshiba, T. (2002) 'On the validity of value-at-risk: Comparative analyses with expected shortfall', *Monetary and Economic Studies*, Vol. 20, No. 1, pp. 57–85.
22 Yamai, Y. and Yoshiba, T. (2002) 'Comparative analyses of expected shortfall and value-at-risk: Their estimation error, decomposition, and optimization', *Monetary and Economic Studies*, Vol. 20, No. 1, pp. 87–121.
23 Artzner, P., Delbaen, F., Eber, J.-M. *et al.* (1999) 'Coherent measures of risk', *Mathematical Finance*, Vol. 9, No. 3, pp. 203–228.
24 Rockafellar, R. T., Uryasev, S. and Zabarankin, M. (2006) 'Generalized deviations in risk analysis', *Finance and Stochastics*, Vol. 10, No. 1, pp. 51–74.
25 Low, C., Pachamanova, D. and Sim, M. (2011) 'Skewness-aware asset allocation: A new theoretical frame-work and empirical evidence', *Mathematical Finance*, Vol. 21, forthcoming.
26 Clarke, R. G. and Arnott, R. D. (1987) 'The cost of portfolio insurance: Tradeoffs and choices', *Financial Analysts Journal*, November/December, Vol. 43, No. 6, pp. 35–47.
27 Leland, H. E. and Rubinstein, M. (1988) 'The evolution of portfolio insurance', in Luskin, D. L. (ed.) 'Portfolio Insurance: A Guide to Dynamic Hedging', John Wiley & Sons, New York, NY, pp. 3–10.
28 Perold, A. F. and Sharpe, W. F. (1988) 'Dynamic strategies for asset allocation',

Financial Analysts Journal, Vol. 44, No. 1, pp. 16–27.
29 Black, F. and Jones, R. (1987) 'Simplifying portfolio insurance', *Journal of Portfolio Management*, Vol. 14, No. 1, pp. 48–51.
30 Estep, T. and Kritzman, M. (1988) 'TIPP: Insurance without complexity', *Journal of Portfolio Management*, Vol. 14, No. 4, pp. 38–42.
31 Bertrand, P. and Prigent, J.-L. (2003) 'Portfolio insurance strategies: A comparison of standard methods when the volatility of the stock is stochastic', *International Journal of Business*, Vol. 8, No. 4.
32 Cintioli, D. and Marchioro, M. (2005) 'Historical-simulation method', Risk International, Yearbook 2005, available at: http://db.riskwaters.com/global/risk/foreign/international/October2005/R_Int_08-11.pdf (accessed 19th January, 2011).
33 Stoxx Euro 50 Total Return (EU0009658178), available at: www.yahoo.com (accessed 20th October, 2010).
34 DJ UBS Commodity (EUR) Total Return (DJUBEUTR), available at: www.djindices.com (accessed 20th October, 2010).
35 Pritsker, M. (2001) 'The hidden dangers of historical simulation', Board of Governors of the Federal Reserve System, Finance and Economic Discussion Series No. 2001-27, July, available at: http://www.federalreserve.gov/pubs/feds/2001/200127/200127pap.pdf (accessed 19th January, 2011).
36 Jorion, P. (2001) 'Value at Risk: The New Benchmark for Managing Financial Risk', McGraw-Hill, New York, NY, p. 223.
37 UniCredit Group Research Portal, available at: http://www.research.unicreditgroup.eu (accessed 20th January, 2011).

ANNEXES: FURTHER RESULTS OF THE CASE STUDY

Annex 1: Frequency distributions of investment strategies

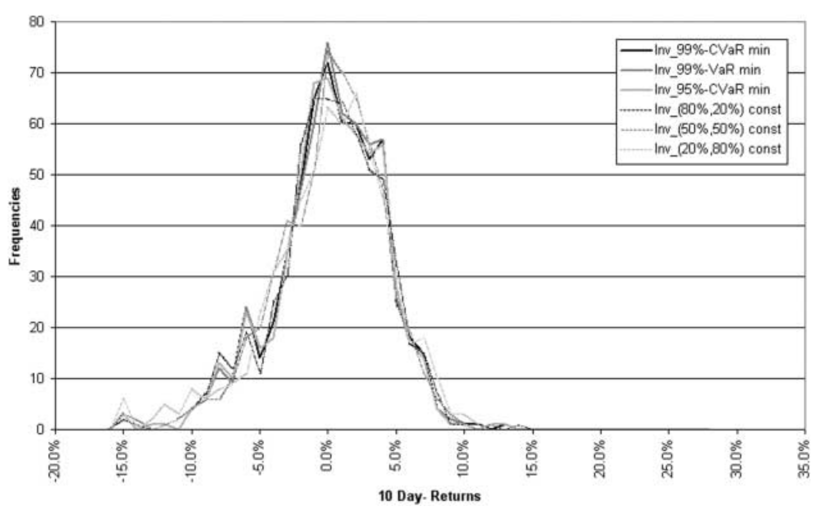

Figure 12: Frequency return distribution of the unsecured investment strategies

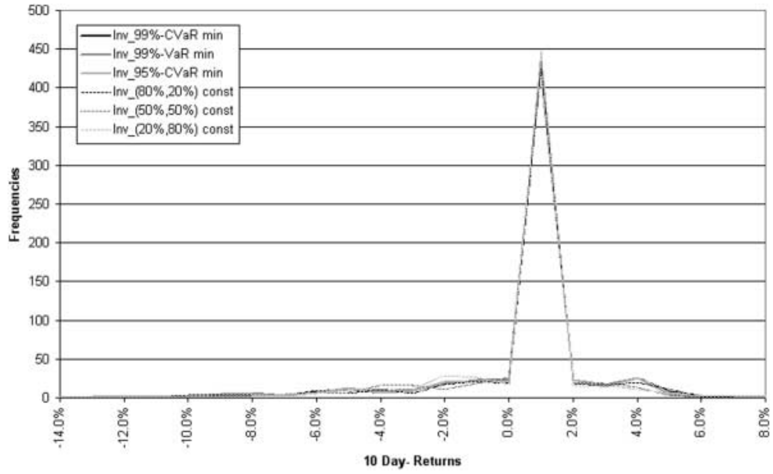

Figure 13: Frequency return distribution of the stop-loss insured investment strategies

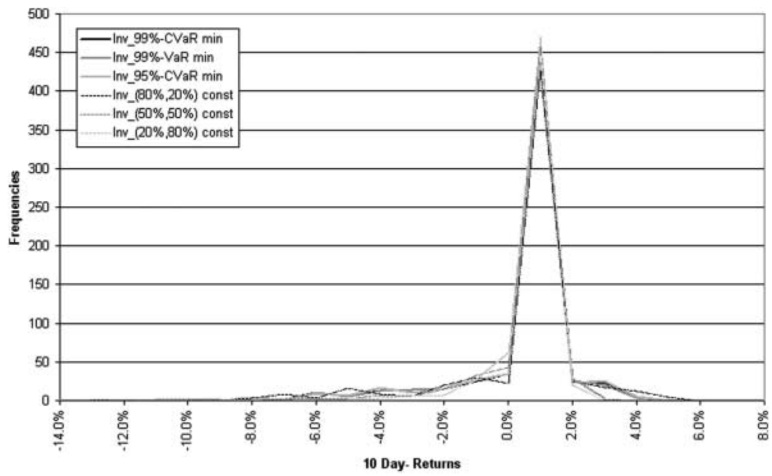

Figure 14: Frequency return distribution of the CPPI insured investment strategies

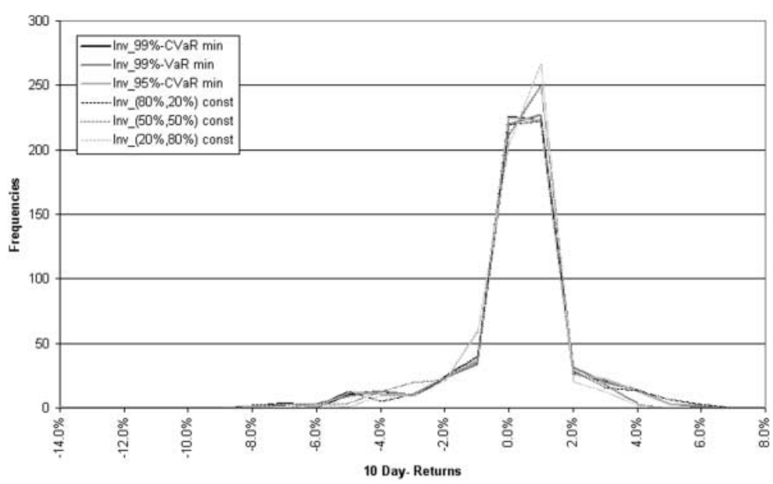

Figure 15: Frequency return distribution of the TIPP insured investment strategies

Annex 2: Risk-return ratios of the distributions

The following tables present the observed risk and return ratios of the 10-day log returns of the different investment strategies with and without portfolio insurance. The first column in every table presents the total performance over the investment horizon (2nd January, 2008 to 31st May, 2010).

Table 9: Risk/return ratios of unsecured investment strategies

Unsecured investment strategies	Total performance	Maximum return	Minimum return	Mean return	93% CVAR of returns	90% CVAR of returns
$Inv_{99\%-CVaR\ min}$	−20.68%	12.38%	−17.65%	−0.39%	−9.10%	−8.30%
$Inv_{99\%-VaR\ min}$	−17.13%	12.52%	−17.38%	−0.32%	−9.02%	−8.21%
$Inv_{95\%-CVaR\ min}$	−21.00%	12.45%	−17.39%	−0.40%	−9.09%	−8.29%
$Inv_{(80\%,20\%)\ const}$	−20.35%	13.11%	−16.29%	−0.38%	−9.30%	−8.46%
$Inv_{(50\%,50\%)\ const}$	−22.21%	11.12%	−19.93%	−0.42%	−9.32%	−8.30%
$Inv_{(20\%,80\%)\ const}$	−25.05%	10.45%	−23.64%	−0.47%	−11.15%	−9.74%

Table 10: Risk/return ratios of investment strategies under stop-loss insurance

Stop−loss secured investment strategies	Total performance	Maximum return	Minimum return	Mean return	93% CVAR of returns	90% CVAR of returns
$Inv_{99\%-CVaR\ min}$	−20.00%	5.70%	−13.31%	−0.36%	−7.24%	−6.00%
$Inv_{99\%-VaR\ min}$	−20.00%	5.46%	−12.90%	−0.36%	−7.10%	−5.90%
$Inv_{95\%-CVaR\ min}$	−20.00%	5.59%	−13.19%	−0.36%	−7.20%	−5.97%
$Inv_{(80\%,20\%)\ const}$	−20.00%	6.01%	−14.68%	−0.36%	−7.68%	−6.35%
$Inv_{(50\%,50\%)\ const}$	−20.00%	5.34%	−9.79%	−0.35%	−5.93%	−5.15%
$Inv_{(20\%,80\%)\ const}$	−20.00%	7.39%	−12.39%	−0.34%	−5.69%	−4.79%

Table 11: Risk/return ratios of investment strategies under CPPI insurance

CPPI secured investment strategies	Total performance	Maximum return	Minimum return	Mean return	93% CVAR of returns	90% CVAR of returns
$Inv_{99\%-CVaR\ min}$	−20.00%	3.82%	−11.03%	−0.36%	−5.69%	−4.77%
$Inv_{99\%-VaR\ min}$	−20.00%	3.63%	−11.10%	−0.36%	−5.60%	−4.70%
$Inv_{95\%-CVaR\ min}$	−20.00%	3.79%	−10.83%	−0.36%	−5.66%	−4.76%
$Inv_{(80\%,20\%)\ const}$	−20.00%	4.97%	−9.28%	−0.36%	−6.10%	−5.13%
$Inv_{(50\%,50\%)\ const}$	−20.00%	2.12%	−13.79%	−0.35%	−5.13%	−4.11%
$Inv_{(20\%,80\%)\ const}$	−20.00%	2.26%	−14.44%	−0.34%	−5.01%	−3.83%

Table 12: Risk/return ratios of investment strategies under TIPP insurance

TIPP secured investment strategies	Total performance	Maximum return	Minimum return	Mean return	93% CVAR of returns	90% CVAR of returns
$Inv_{99\%-CVaR\ min}$	−12.89%	5.26%	−8.38%	−0.22%	−4.72%	−4.00%
$Inv_{99\%-VaR\ min}$	−12.27%	5.03%	−8.17%	−0.21%	−4.64%	−3.95%
$Inv_{95\%-CVaR\ min}$	−12.93%	5.17%	−8.28%	−0.22%	−4.68%	−3.97%
$Inv_{(80\%,20\%)\ const}$	−10.90%	5.74%	−8.63%	−0.19%	−4.73%	−3.95%
$Inv_{(50\%,50\%)\ const}$	−16.29%	3.48%	−7.61%	−0.28%	−4.32%	−3.73%
$Inv_{(20\%,80\%)\ const}$	−17.22%	3.70%	−9.64%	−0.29%	−3.92%	−3.29%

Annex 3: Portfolio insurance application on 99%-CVaR-minimising and 99%-VaR-minimising investment strategies

Stop-loss insurance

Figures 16 and 17 illustrate the application of the stop-loss insurance on the 99%-CVaR- and 99%-VaR-minimising investment strategies. On 10th October, 2008, the risky portfolio for both strategies is shifted into the cash position.

Both figures illustrate that under stop loss the upside market potential in April/May 2010 is not captured by this insurance strategy.

CPPI insurance

Figures 18 and 19 illustrate the application of CPPI insurance on the 99%-CVaR- and 99%-VaR-minimising investment strategies. The cash position is increased since the first half of 2008. For both strategies, the portfolio is completely shifted into the riskless asset when the floor level is reached on 14th October, 2008.

Figures 18 and 19 illustrate that CPPI does not capture upside market potential in the recovering market.

TIPP insurance

Figures 20 and 21 illustrate the application of TIPP insurance on

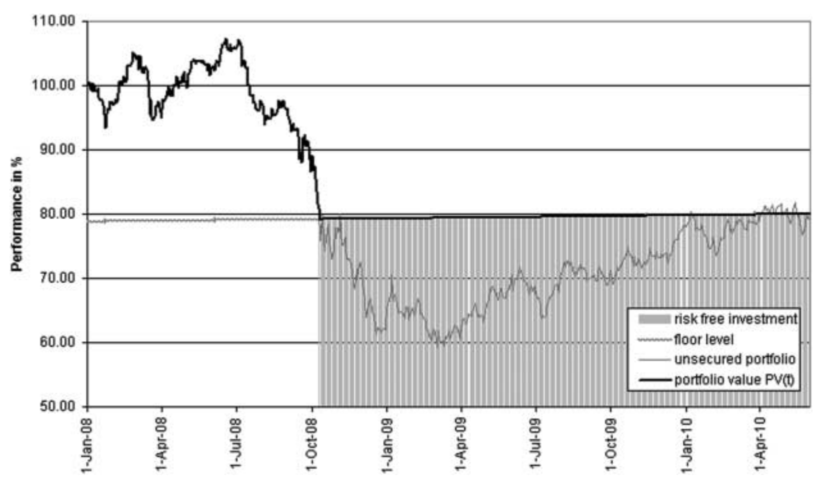

Figure 16: Stop-loss insurance on the 99%-CVaR-minimising investment strategy

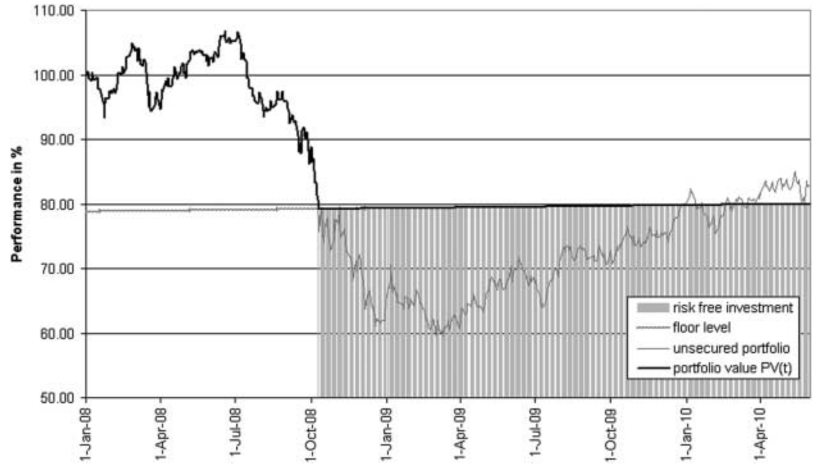

Figure 17: Stop-loss insurance on the 99%-VaR-minimising investment strategy

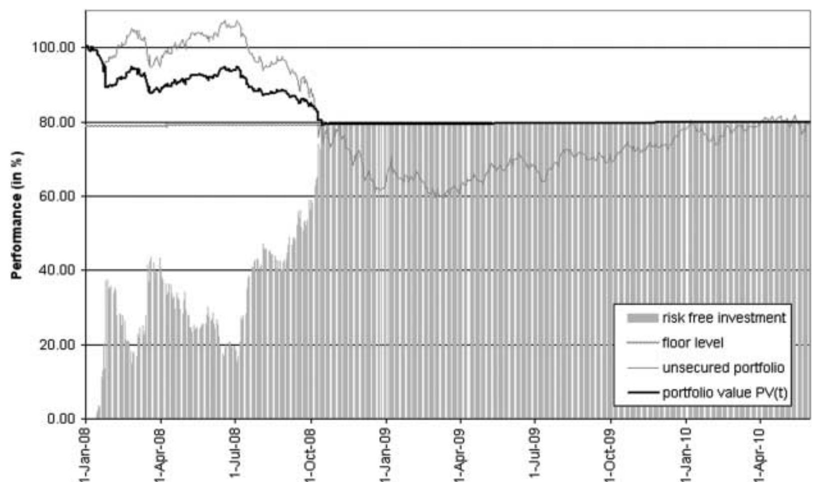

Figure 18: CPPI insurance on the 99%-CVaR-minimising investment strategy

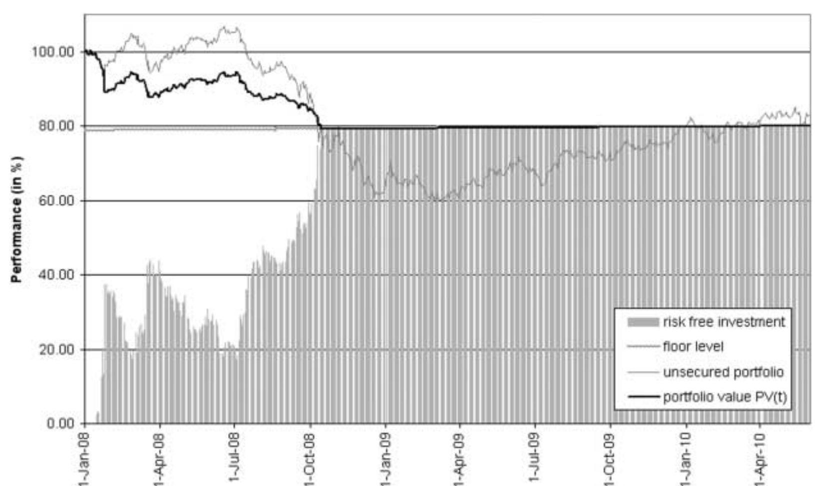

Figure 19: CPPI insurance on the 99%-VaR-minimising investment strategy

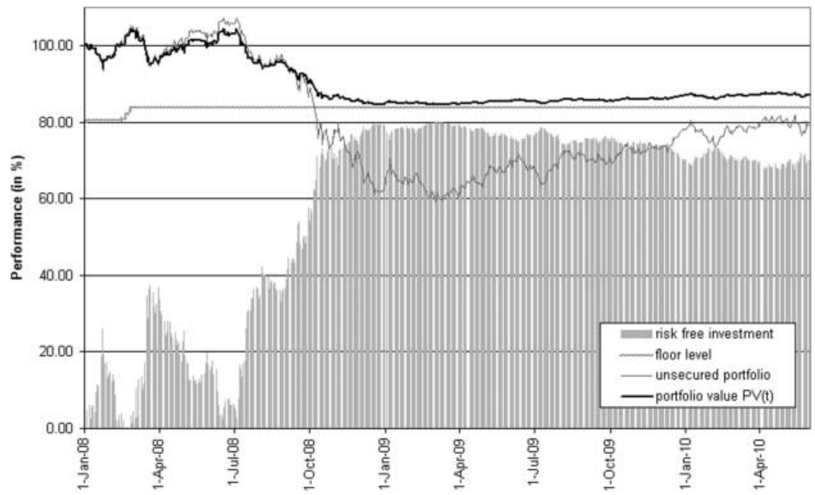

Figure 20: TIPP insurance on the 99%-CVaR-minimising investment strategy

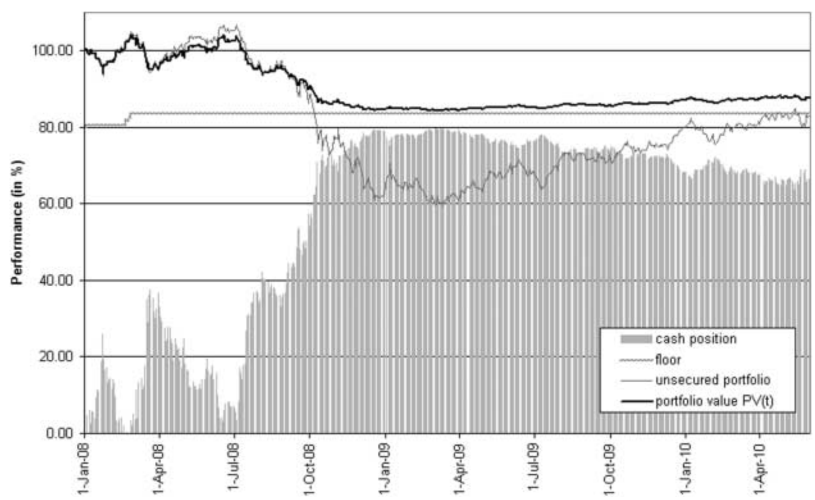

Figure 21: TIPP insurance on the 99%-VaR-minimising investment strategy

the 99%-CVaR- and 99%-VaR-minimising investment strategies. The cash position is intensely increased from the first half of 2008 and gradually reduced starting in the first half of 2009 for both strategies.

The progressions of TIPP both strategies illustrate that upside potential is captured under TIPP insurance.