Risk-Return Optimization of the Bank Portfolio

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Abstract

In an intensifying competition banks are forced to develop and implement enterprise wide integrated risk-return management systems. Financial risks have to be limited and managed from a bank wide portfolio perspective. Risk management rules must be accomplished from internal and regulatory points of view. Expected returns need to be maximized subject to these constraints, leading to a generalized portfolio optimization problem under different capital limits.

We give a survey on a risk-return optimization model for the bank portfolio that maximizes the expected returns to the planning horizon with respect to internal and regulatory loss risk constraints. We derive consistent planning information that ensures efficient return targets and maximal capital use of the economic and the regulatory capital. The impact of the optimization is shown by an application example.

1 Introduction

In an intensifying competition banks are forced to develop and implement enterprise wide integrated risk-return management systems. Financial risks have to be limited and managed from a bank wide portfolio perspective. Risk management rules must be accomplished from internal and regulatory points of view. Expected returns need to be maximized subject to these constraints, leading to a generalized portfolio optimization problem under different capital limits.

In chapter 2 we give a survey on a risk-return optimization model that maximizes the expected returns of the bank portfolio to the planning horizon subject to internal and regulatory loss risk ceilings. The internal risk constraint is based on the new risk measure of Conditional Value at Risk (CVaR), that has been proved to be appropriate for measuring bank wide loss risk [4,5]. We solve the optimization problem by a CVaR-optimization algorithm by Rockafellar/Uryasev [4,5]. The regulatory capital restrictions represent the 'Basle Rules' of risk limitation [1,2]. We derive consistent planning information from the optimum solution that ensures efficient return targets and a maximal use of the economic and regulatory capital. The impact of the optimization is shown by an application example in chapter 3. We close by a brief summary in chapter 4.

2 Risk-Return Optimization Model for the Bank Portfolio

2.1 Survey

For its planning processes the bank needs to identify risk-return efficient target portfolios, that maximize expected returns to the planning horizon and meet risk constraints from different points of view. From an internal perspective, the bank limits its loss risks by the economic capital available. At the same time, the bank must observe legal loss risk boundaries of the 'Basle rules', that comprise constraints on the risks of the banking book and the trading book, combined with limits on the capital components that cover the different kinds of risks [1,2,7]. We achieve the following general model structure of the bank wide risk-return portfolio optimization problem (P):

(P) (1)
Objective function: Maximize expected returns
subject to constraints
Constraint 1: Internal risk ≤ Economic capital,
Constraint 2: Regulatory risk ≤ Regulatory capital,
Constraints on the regulatory capital components,
Constraint 3: Position bounds, definition of the feasible solutions.

We develop the optimization model in three steps. First we define a risk measure that is appropriate to measure the loss risk of the bank portfolio (chapter 2.2). Next we introduce an optimization algorithm for the solution of the basic risk-return optimization problem with respect to the constraints 1 and 3 (chapter 2.3). We then extend the problem to an optimization model for the bank planning process by integrating the regulatory risk constraints (chapter 2.4).

2.2 Definition of the CVaR Risk Measure

While the risk measure of Value at Risk, commonly applied in finance for market risk measurement, lacks the elementary property of sub-additivity, if the loss distributions are not normal, the *Conditional Value at Risk* (CVaR), defined as the conditional expectation beyond the Value at Risk, has been proved to be appropriate for risk measurement of any loss distributions [4,5].

Let $\mathbf{x}$ be the vector of the positions of the bank portfolio assets and $\mathbf{y}$ the vector of the corresponding market prices. We define the Conditional Value at Risk deviation of the portfolio loss $\mathrm{CVaR}_\alpha(L(\mathbf{x},\mathbf{y}))$ as

$$\mathrm{CVaR}_\alpha(L(\mathbf{x},\mathbf{y})) = E[L(\mathbf{x},\mathbf{y}) \mid L(\mathbf{x},\mathbf{y}) \geq \mathrm{VaR}_\alpha(L(\mathbf{x},\mathbf{y}))], \quad (2)$$

where $L(\mathbf{x},\mathbf{y})$ is defined as the difference of the uncertain from the expected portfolio value at the horizon, $L(\mathbf{x},\mathbf{y})=E[\mathbf{y}]'\mathbf{x}-\mathbf{y}'\mathbf{x}$, and $\mathrm{VaR}(L(\mathbf{x},\mathbf{y}))$ is the α-quantile of the loss function L(**x,y**).[1] In the case of discontinuities at the α-quantile, CVaR can be defined as a weighted average of the VaR and the conditional expectation beyond the VaR [5].

In the above definition CVaR deviation is a convex risk measure that ensures the existence of a risk minimum portfolio on a convex set and the solvability of the optimization problem (P). It is appropriate to measure loss risk from any asymmetric and discontinuous loss distribution with discrete probabilities [5]. The following figure shows the loss function and the risk measure of CVaR deviation at the confidence level α:

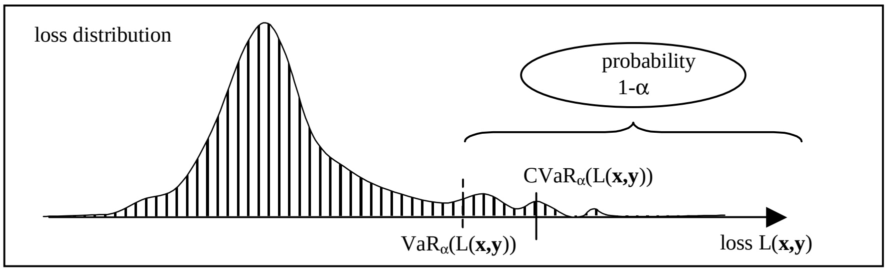

Fig. 1. Loss distribution L(**x,y**) and risk measures VaR and CVaR

2.3 Basic Risk-Return Optimization Model

We introduce an optimization algorithm that maximizes expected returns with respect to a CVaR risk constraint that can be applied to portfolios with any loss distribution. We make use of this approach to solve the basic problem of the optimization model (P) to maximize the expected portfolio return subject to the constraints 1 and 3.

Let $\mathbf{x} = (x_1,\ldots,x_n)'$ be the decision variable, i.e. the positions of the single assets. We define a linear objective function for the expected portfolio return $\mu(\mathbf{x})=\boldsymbol{\mu}'\mathbf{x}$, with $\boldsymbol{\mu}=(\mu_1,\ldots,\mu_n)'$ the vector of the expected returns of single assets. The internal loss risk is measured by CVaR deviation of the loss function L(**x,y**) and constrained by the maximum amount of economic capital, denoted as ec_cap_max. The area of the feasible solutions is defined by upper and lower position bounds, the vectors **low_bound** and **up_bound** respectively. We solve this generalized risk-return portfolio optimization problem (P) by an algorithm of Rockafellar/Uryasev [4,5]. Based on a scenario generation $\mathbf{y_1}, \ldots, \mathbf{y_K}$ of the market prices of the portfolio assets, the CVaR-constraint is approximated by a set of linear constraints, leading to a linear optimization problem. We achieve the following basic optimization model (P_{CVaR}) that maximizes the expected portfolio returns with respect to the constraints 1 and 3 of the optimization problem (P):

[1] We apply the term *CVaR deviation*, as the loss distribution measures the deviation of the uncertain from the expected portfolio value.

(P_{CVaR}) (3)

Objective Function $\mu(\mathbf{x}) = \boldsymbol{\mu}'\mathbf{x} = \sum_{j=1}^{n} \mu_j x_j$,

Constraint 1: Internal Risk Constraint

(i) $q + \frac{1}{(1-\alpha)} \cdot \frac{1}{K} \sum_{k=1}^{K} z_k \le ec\_cap\_max,$

(ii) $L(\mathbf{x}, \mathbf{y_k}) - q \le z_k, k = 1, \dots, K,$

(iii) $-z_k \le 0, k = 1, \dots, K,$

(iv) $q \in \Re.$

"Internal loss risk (CVaR deviation estimate) ≤ Economic capital"

Constraint 3: Boundaries of the Feasible Solutions

(vi) $\mathbf{low\_bound} \le \mathbf{x} \le \mathbf{up\_bound}.$

2.4 Extension to an Optimization Model for the Bank Portfolio

We extend the basic optimization model (P_{CVaR}) to an optimization model (P) for the bank planning processes. Beneath the internal loss risk boundaries the bank has to comply with regulatory rules of risk limitation passed by the Basle Committee on Banking Supervision [1,2,7].[2] We give a survey how these constraints are integrated into the optimization model [6].

The *bank book positions*[3] are restricted with respect to their credit risk by a linear constraint. The sum of the risk weighted assets is limited by the regulatory capital resources of the core ('tier 1') capital plus the supplementary ('tier 2') capital. Capital charges of the *trading book* are based on the general market risk and the specific risk, as well as the counterparty risk of the trading book positions. We model linear constraints for the specific and the counterparty risk. With respect to the general market risk of the trading book we assume that the bank applies an internal market risk model to estimate the Value at Risk with the 'Basle parameters' [2,7]. As CVaR represents an upper bound of the corresponding VaR of the loss distribution, we apply a CVaR constraint on the general market risk of the trading book to achieve an upper bound of the regulatory VaR. We limit the sum of the general market risk, measured by the right hand side of the trading book CVaR constraint, the specific and the counterparty risk by the applicable regulatory capital components, that consist of the tier 1 and tier 2 capital elements that are not used to cover bank book risks and the tier 3 capital[4] available. Further we model constraints that limit the regulatory capital components [1,2,6].

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[2] We consider the prevailing regulatory risk limitation rules. The model allows a transition to the new 'Basle II' rules [3], that will require different input data, but will not influence the basic structure of the risk constraints.

[3] In brief, the bank book comprises all 'non trading' assets, while the trading book comprises all positions the bank is holding for trading purposes (precise definition see [2]).

[4] The tier 3 capital mainly comprises subordinate short term debt [2,7].

3 Application Example

We illustrate the effects of the risk-return optimization by a simplified application example [6]. An XY-Bank consists of four typical bank assets: asset 1 represents high quality bank bonds (rating AA), asset 2 corporate bonds (rating A), asset 3 industrial loans (rating B) and asset 4 a trading portfolio that is dependent on an equity index. In the actual situation, the regulatory capital of the bank is used at 93.80% and cannot be increased in the next business year. The initial portfolio uses 76.20 units of economic capital.

In order to gain additional profits, the managing board considers to raise the economic capital, i.e. the level of internal risk by additional undisclosed reserves of 17.10 units. The managing board wants to know, if the additional economic capital will lead to higher returns in the next business year and will be suitable to improve the portfolio risk-return relations and to meet the internal hurdle-rate of an overall *return on risk adjusted capita*l (RORAC), that is defined as the expected portfolio return divided by the portfolio CVaR deviation, of 14.00%.

We apply the optimization model (P) with increasing CVaR levels to generate the efficient line as described in the Fig. 2 below. The initial portfolio is denoted by PF 0, the optimal portfolio at the given level of risk by PF 1 and the optimal portfolio with the increase of the economic capital by PF 2.

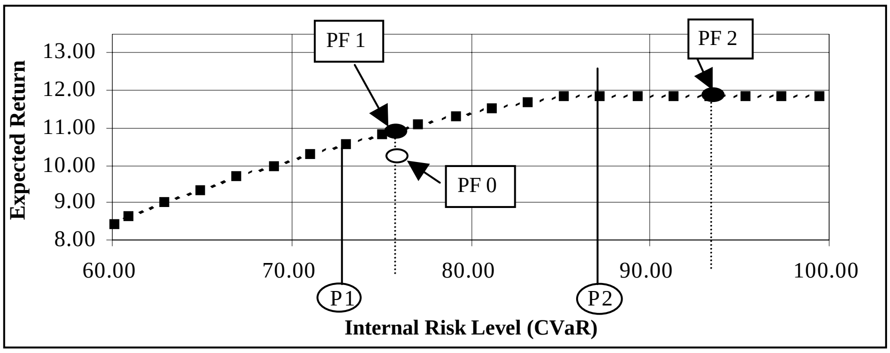

Fig. 2. Efficient Line of the XY-Bank Portfolio Resulting of Applications of (P)

Analyzing the efficient line, we observe that to the left of the CVaR level P1 only the internal risk constraint is active and the regulatory capital is not completely used. To the right of the CVaR level P2 only the regulatory constraint is active, as in this interval we achieve a stationary solution due to the regulatory capital constraint and the economic capital is not totally devoted. In the interval [P1,P2] both capital constraints are active, i.e. both capital resources, the economic and the regulatory capital, are maximal utilized.

A comparison of the optimal portfolios PF1 and PF 2 shows that the increase of the economic capital leads to higher expected returns, however the RORAC of PF 2 of 13.57% does not meet the hurdle rate of 14.00% and is even lower than the RORAC of the initial portfolio PF 0 of 13.98%. Also, the economic capital constraint is not active and the economic capital cannot be maximal used in the portfolio PF 2.

We deduce that an increase of the economic capital is not advisable and that the actual level of risk should be maintained. The expected return of the portfolio PF 1 improves the expected return of the initial portfolio PF 0 by 0.31 units at the given level of internal risk. Its portfolio RORAC of 14.39% complies with the internal hurdle rate. Both capital resources, the economic and the regulatory capital, can maximal be used. Analyzing the risk-return relations along the efficient line we find that a maximal RORAC can be achieved in the interval [68.9,71.0]. However, the implementation of a RORAC optimizing strategy would require to reduce absolute volumes and expected returns. This might be conflicting with other corporate goals and may not be supported by the shareholders.

4 Conclusion

We have introduced a risk-return optimization model for the bank portfolio, that can be applied in the planning processes of the bank in order to identify risk-return efficient target portfolios. It maximizes the expected returns subject to internal and regulatory risk constraints. It is based on the new risk measure of CVaR, which has is appropriate for bank wide portfolio risk measurement. The optimization problem can be solved by linear programming techniques. The optimization model generates consistent planning information. It spots risk-return efficient portfolios and finds intervals of maximal use of both capital resources, the available economic and regulatory capital, and of highest portfolio RORACs, thus providing basic information for a bank wide risk-return management process and contributing to an enhancement of the competitive position of the bank.

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