Regulatory Impacts on Credit Portfolio Management

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Abstract

Efficient credit portfolio management is a key success factor of bank management. Discussions of the new capital adequacy proposals by the Basle Committee on Banking Supervision enlighten the necessity to consider the credit risk management both from the internal and the regulatory point of view. We introduce an optimization approach for the credit portfolio that maximizes expected returns subject to internal and regulatory risk constraints. With a simplified bank portfolio we examine the impact of the regulatory risk limitation rules on the optimal solutions.

1 Introduction

Efficient credit portfolio management is a key success factor of bank management. In an adverse market environment and intensifying competition banks are exposed to increasing risks and decreasing return margins of their credit portfolio, while bank shareholders are demanding higher risk premiums for their invested capital. The ability to identify risk-return optimal portfolios becomes a fundamental element of credit portfolio management. The recent discussions of the Basle Committee on Banking Supervision enlighten the necessity to manage credit risk simultaneously from an internal and a regulatory perspective.

In this paper, we give a survey of a new optimization algorithm that determines risk-return efficient credit portfolios under internal and regulatory credit risk constraints. We formulate the optimization problem for the credit portfolio based on the new risk measure, Conditional Value at Risk, and derive risk-return ratios for the optimal portfolios (chapter 2). With an application example, we analyze the risk-return structure of an optimal portfolio. We examine the impact of the regulatory risk limitation rules and visualize how they may lead to inefficiencies in the credit portfolio management (chapter 3).

2 Optimization Approach

2.1 Definition of the CVaR Risk Measure

The risk measure Value at Risk (VaR), commonly applied in finance, lacks the sub-additivity property, when return distributions are not normal. This means that the diversification of the portfolio may increase portfolio VaR. A similar percentile risk measure, *Conditional Value at Risk* (CVaR) does not have this drawback. The term Conditional Value-at-Risk was introduced in [5]. For continuous distributions, CVaR is equal to the conditional expectation beyond VaR, see [5]. However, for general distributions, it is a weighted average of VaR and the conditional expectation beyond VaR, see [6]. CVaR can be applied to measure loss risk from any asymmetric and discontinuous loss distribution with discrete probabilities and it obeys the property of coherence, see [1,4,6], a set of axioms that a risk measure should meet from the point of view of a regulator [2]. CVaR has been proved to be appropriate for credit portfolio risk measurements [4,5,6].

Let $\mathbf{x}=(x_1,\ldots,x_n)'$ be a vector of positions of credit assets of a portfolio, and $\mathbf{y}=(y_1,\ldots,y_n)'$ be a vector of the corresponding market prices. For continuous distributions, we define *CVaR deviation* $\mathrm{CVaR}_\alpha^\Delta(L(\mathbf{x},\mathbf{y}))$ of the portfolio loss risk as

$$\mathrm{CVaR}_\alpha^\Delta(L(\mathbf{x},\mathbf{y})) = E[L(\mathbf{x},\mathbf{y}) \mid L(\mathbf{x},\mathbf{y}) \geq \mathrm{VaR}_\alpha(L(\mathbf{x},\mathbf{y}))], \quad (1)$$

where the loss function $L(\mathbf{x,y})$ is the difference of the uncertain portfolio values and the expected value of the portfolio, i.e. $L(\mathbf{x,y})=E[\mathbf{y}]'\mathbf{x}-\mathbf{y}'\mathbf{x}$, and $\mathrm{VaR}(L(\mathbf{x},\mathbf{y}))$ is the α-quantile of the loss function $L(\mathbf{x,y})$.[1]

2.2 Formulation of the Optimization Model

The optimization problem models the basic goal of the credit portfolio management. We maximize the expected portfolio return $\mu(\mathbf{x})=\boldsymbol{\mu}'\mathbf{x}$ under internal and regulatory loss risk constraints [8], with $\mathbf{x}$ the decision vector and $\boldsymbol{\mu}=(\mu_1,\ldots,\mu_n)'$ the vector of the expected returns of single assets. The internal loss risk is measured by the CVaR deviation of the portfolio loss according to equation (1) and is constrained by the maximal amount of economic capital available, denoted as ec_cap_max. Based on the optimization algorithm of Rockafellar/Uryasev [5], the CVaR constraint is approximated by a set of linear constraints, leading to a linear optimization problem. To implement the algorithm, as input data, we use a sample of market price scenarios $\mathbf{y_1}, \ldots, \mathbf{y_K}$ of the vector $\mathbf{y}$.[2] The regulatory credit risk is measured by the regulatory risk based capital ratios, **reg_cap** = $(\text{reg\_cap}_1,\ldots, \text{reg\_cap}_n)'$ and is limited by the available regulatory core and supplementary capi-

[1] In the case of nonzero probability atom at the α-quantile, CVaR is defined as the weighted average of VaR and the conditional expectation beyond the VaR [6].

[2] In the application example in the next chapter, these market price scenarios are generated by a Monte Carlo-Simulation according to the CreditMetrics approach of J. P. Morgan.

tal, denoted by reg_cap_max. The area of the feasible solutions is defined by upper and lower position bounds, the vectors **low_bound** and **up_bound**. We solve the following linear optimization model:

Objective Function $\mu(\mathbf{x}) = \boldsymbol{\mu}'\mathbf{x} = \sum_{j=1}^{n} \mu_j x_j,$ (4)

Constraint # 1: Internal Risk Constraint

(i) $q + \frac{1}{(1-\alpha)} \cdot \frac{1}{K} \sum_{k=1}^{K} z_k \le ec\_cap\_max,$

(ii) $L(\mathbf{x},\mathbf{y_k}) - q \le z_k, k = 1,...,K,$

(iii) $-z_k \le 0, k = 1,...,K,$

(iv) $q \in \Re.$

"Internal loss risk (CVaR deviation estimate) ≤ Economic capital"

Constraint # 2: Regulatory Risk Constraint

(v) **reg_cap'x** ≤ reg_cap_max.

Constraint # 3: Boundaries of the Feasible Solutions

(vi) **low_bound ≤ x ≤ up_bound.**

In order to analyze the effects of the regulatory risk constraints on the optimal portfolios, we consider the following optimization models (P') and (P) with and without the regulatory risk constraint, accordingly:

(P): *Maximize* Objective *subject to* Constraints # 1 and 3,
(P'): *Maximize* Objective *subject to* Constraints # 1, 2, 3. (5)

2.3 Risk-Return Analysis of the Portfolio Assets

The contributions of the single assets to the overall portfolio risk and return represent basic information for the risk-return analysis of the optimal portfolios. The return contribution $\mu_j(\mathbf{x})$ of the j-th asset to the portfolio $\mathbf{x}$ is given by the j-th term of the return function, i.e. $\mu_j(\mathbf{x})=\mu_j x_j$, j=1,…,n.

We apply the Euler allocation principle to derive the risk contributions of the single assets [3,7]: The risk contribution $r_j(\mathbf{x})$ of the j-th asset is based on the partial derivative of the portfolio risk measure with respect to the j-th asset. It corresponds to the conditional expected loss of the j-th component in the tail of the portfolio loss distribution and can be estimated from the given sample of the market prices as the mean of the losses of the j-th asset in the tail of the loss distribution [7]. We achieve the following risk contribution $r_j(\mathbf{x})$ of the j-th asset, j=1,…,n:

$$r_j(\mathbf{x}) = \frac{\partial\, CVaR_{\alpha}^{\Delta}(L(\mathbf{x},\mathbf{y}))}{\partial\, x_j} \cdot x_j = E[L_j(\mathbf{x},\mathbf{y}) \mid L(\mathbf{x},\mathbf{y}) \ge VaR_{\alpha}(L(\mathbf{x},\mathbf{y}))], \qquad (6)$$

where $L_j(\mathbf{x},\mathbf{y}) = E[y_j]x_j - E[y_j x_j \mid L(\mathbf{x},\mathbf{y}) \ge VaR_{\alpha}(L(\mathbf{x},\mathbf{y}))], j = 1,...,n.$

We define the risk-return ratios of single assets, the return on risk adjusted capital $RORAC_j(\mathbf{x})$ of the j-th asset and the return on equity $RoE_j(\mathbf{x})$, i.e. the return on the regulatory capital, of the j-th asset as

$$\text{(i) } \mathrm{RORAC}_j(\mathbf{x}) = \frac{\mu_j(\mathbf{x})}{r_j(\mathbf{x})}, \qquad j = 1,\dots,n, \tag{7}$$

$$\text{(ii) } \mathrm{RoE}_j(\mathbf{x}) = \frac{\mu_j}{reg\_cap_j}, \qquad j = 1,\dots,n.$$

3 Application Example

An ABC Bank consists of three typical credit assets: asset 1 represents high quality bonds (Rating AA), asset 2 mortgage loans (Rating BB) and asset 3 retail loans (Rating B). 10 units of regulatory capital are available, of which 94% are actually in use. The internal risk (CVaR) level may be varied to some extent according to the risk policy of the bank. The initial portfolio uses 48 units of the economic capital. Our goal is to investigate how the risk-return relations of the initial credit portfolio can be improved and how the regulatory risk constraint effects the optimal portfolios. We applied the optimization models (P) and (P') with different CVaR levels. First, we generated the efficient frontiers and analyzed the overall portfolio risk-return relations. Next, we analyzed the risk-return structures of the single assets of the optimal portfolios.

As shown in the Fig. 1, we observe that the regulatory constraint becomes active at the CVaR-level of 39.9 units. At the given capital levels (ec_cap_max=48, reg_cap_max=10), the expected portfolio returns can be improved by 0.07 units in (P') and by 0.23 units in (P). This means that without the regulatory constraint an additional profit of 0.16 units could be gained. The portfolio RORAC, defined as the expected return $\mu(\mathbf{x})$ divided by the CVaR deviation $\mathrm{CVaR}_{\alpha}^{\Delta}(L(\mathbf{x},\mathbf{y}))$ of the portfolio $\mathbf{x}$, increases from 6.09% to 6.29% in (P') and to 6.63% in (P).

We also observe that the ABC bank can generate higher portfolio RORACs by lowering the level of internal risk. The maximal RORAC of 6.82% can be reached at the interval of ec_cap_max=[34.9,37.7], where the regulatory constraint is not active. However, the implementation of a RORAC optimizing strategy would require reducing the credit volumes and absolute returns. This might be conflicting with other corporate goals and may not be supported by the shareholders.

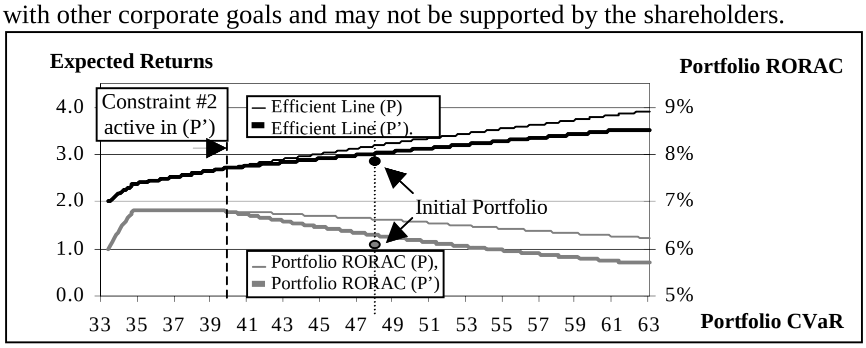

Fig. 1. Efficient Lines and Portfolio RORACs of the Optimization Problems (P) and (P')

In order to analyze the risk-return structure of optimal portfolios we first examine the positions of single assets, which are represented in fig. 2. The narrow and broad lines represent the positions of single assets in the solutions of (P) and (P'), respectively. Starting from the minimal CVaR portfolio, the assets are increased in the optimal solutions in the order of descending RORACs, as defined in the equ. 7 (i).[3] When the regulatory constraint becomes active, we observe the effect of capital arbitrage: assets with higher RoEs are preferred to assets with higher RORACs, and the overall portfolio level of risk is increased.

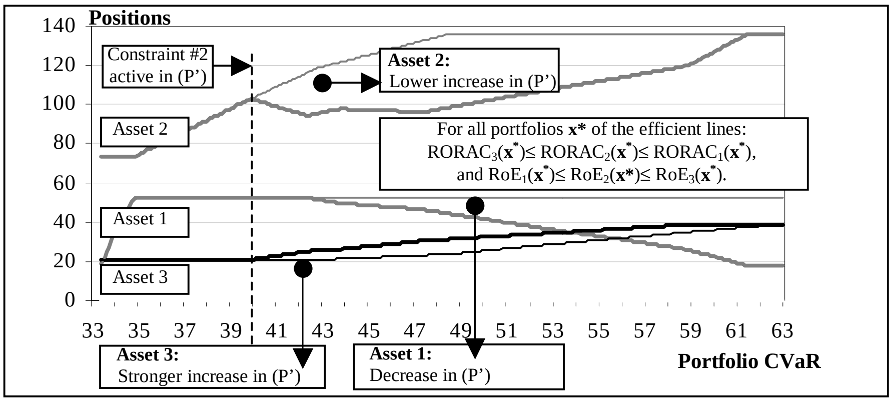

Fig. 2. Impact of the Regulatory Constraint on the Optimal Portfolio Structures

In order to analyze the effect of capital arbitrage more closely, we examine the risk-return structure of the optimal portfolio at the initial CVaR level of 48 units, as described in the fig. 3. Without the regulatory risk constraint, position of asset 1 with highest RORAC is increased by 50%, of asset 2 by 28.3% and position of asset 3 with lowest RORAC is reduced by 21.5%. In (P') asset 1, showing the lowest RoE, is increased less than in (P). Position of asset 3 with the highest RoE is increased, while position of asset 2 with higher RORAC but lower RoE than asset 3 is reduced. The riskier assets are weighted higher in (P'), resulting in lower returns at the given CVaR-level and a sub optimal use of the economic capital, as could be observed in the fig. 1 above.

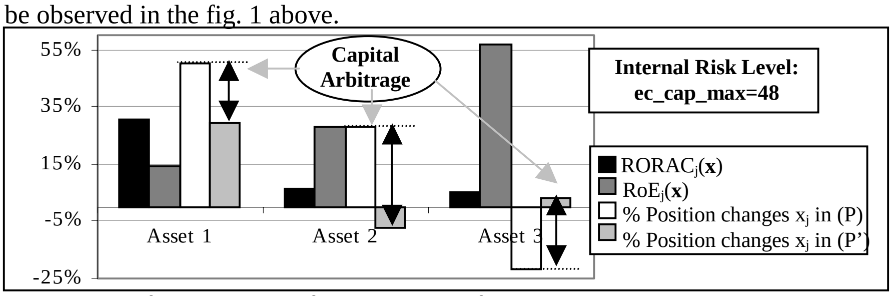

Fig. 3. Portfolio structures of the optimal portfolio in (P) and (P')

[3] Although the RORACs of the single assets of the optimum portfolios **x*** differ slightly along the efficient line, their ranking remains constant, i.e. $RORAC_3(\mathbf{x}^*) \le RORAC_2(\mathbf{x}^*) \le RORAC_1(\mathbf{x}^*)$.

4 Conclusion

We have introduced an algorithm that maximizes the expected returns of a credit portfolio subject to the internal and regulatory risk constraints. It is based on the new risk measure, CVaR, which is appropriate for credit portfolio risk measurement and can be solved by linear programming methods. The optimization model allows to spot intervals of efficient use of both capital resources, the available economic and regulatory capital, and of highest portfolio RORACs. It identifies "unrealized" profits due to the regulatory risk constraint. We conducted risk-return analyses of single assets of the optimal portfolios and found evidence of capital arbitrage, that leads to sub optimal portfolios under the regulatory risk limitation rule, as assets of higher RoE but higher risk are weighted higher than assets of lower risk and higher RORACs.

In a follow-up study we will pursue the application example for the Basle II Accord. We will analyze the impact of new risk weights on optimal credit portfolios. Further, we will investigate how the internal and regulatory risk-return structures of the single assets influence the optimal solutions when both capital constraints are active. Also, the intuitive statement that the single assets are increased in the optimum solutions in the order of descending RORACs in (P) can be formally investigated, as the risk contributions are not explicitly modeled in the optimization algorithm. Another point of interest is to develop an optimization algorithm that calculates the RORAC-optimal portfolios of (P) and (P') in one optimization run.

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